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The Intergenerational State Education and Pensions

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When credit markets to finance investment in human capital are missing, the competitive equilibrium allocation is inefficient. When generations overlap, this failure can be mitigated by properly designed social arrangements. We show that public financing of education and public pensions can be designed to implement an intergenerational transfer scheme supporting the complete market allocation. Neither the public financing of education nor the pension scheme we consider resemble standard ones. In our mechanism, via the public education system, the young borrow from the middle aged to invest in human capital. They pay back the debt via a social security tax, the proceedings of which finance pension payments. When the complete market allocation is achieved, the rate of return implicit in this borrowing–lending scheme should equal the market rate of return.

1. INTRODUCTION

A well-established tradition in public economics argues that government policies and, in particular, most institutions comprising the welfare state are justified by the inability of decentralized markets to deliver a Pareto efficient allocation. This approach allows one to afford both a positive and a normative perspective. It explains the existence of certain arrangements as cooperative remedies to allocational inefficiencies, while also providing guidance to the optimal design of such institutions.

In this paper we adopt a normative stance, and study the role of public education and public pensions. We build a simple dynamic environment in which the lack of a specific credit market leads to a suboptimal accumulation of human capital. By construction, public financing of education is desirable. One may be led to think that this is, in fact, all that it is required to re-establish efficiency. We show that, in general, this is not the case. Introducing a scheme for the public financing of education need not, by itself, restore either the complete market allocation or economic efficiency. An additional institutional arrangement, closely resembling a public pension system, is also needed. Further, we show that a simple but so far altogether ignored linkage between the two systems must hold. This is formally captured by the risk-adjusted equality between the two rates of return implicit in the public financing of education and pensions and the market rate of return on capital. As these can be quite precisely measured in the data, our normative considerations are not empty, as one can effectively measure how far real world welfare systems are from the efficient one. For this reason, after characterizing the optimal intergenerational arrangement we briefly discuss some practical ways of implementing it.

In an earlier paper, Becker and Murphy (1988) argue informally that the welfare state serves purposes previously served by intra-family arrangements, which were instrumental for implementing efficient intergenerational allocations. We examine this conjecture in the context of a well specific dynamic general equilibrium model of human capital accumulation. Young generations would like to accumulate productive human capital, but are unable to finance it via credit markets. Middle age individuals would like to diversify their retirement portfolios by investing in the human capital of younger people, but financial instruments to do so are unavailable. In such circumstances it is often argued that parental altruism and within family arrangements may compensate for the missing financial instruments, thereby greatly reducing the scope for public intervention. In principle, we share this view; nevertheless, two considerations should be taken into account. Efficiency is achieved only when parents fully internalize the utility of all future generations, and are not credit constrained in turn. As the abundant empirical literature on bequests has convincingly shown, this does not appear to be the case, at least in general. Further, and on pure theoretical grounds, the ability of within family arrangements to replicate the complete market allocation (*e.g.* Kotlikoff and Spivak, 1981) may be severely affected by the same enforcement and incentive problems that are commonly believed to make private markets for financing human capital investment hard to sustain.

The model we study is very stylized, but its main implications are robust to the addition of more realistic features. In particular, introducing population growth and a realistic number of periods of life leave the results unaltered (Boldrin and Montes, 2001). Adding some form of parental (or filial, as in Boldrin and Jones, 2002) altruism would modify the quantitative but not the qualitative prescriptions, unless one adopts the fully dynastic model of familial relations of Barro and Becker (1989). Adding uncertainty, in the form of unexpected shocks to the productivity of either human or physical capital, would most likely strengthen our normative prescriptions on the grounds of portfolio diversification. This is akin to the point already made, long ago, by Merton (1983) in a different context, but with similar implications for policy. Finally, and aside from the redistributive concerns this may or may not create for public policy, the introduction of heterogeneity within a generation would also not alter our main prescription. Different results would most likely follow from the introduction of endogenous life-cycle labour supply, which would be affected by changing opportunities for financing education. This is an important extension we find worth pursuing.

Neither Becker and Murphy (1988) nor we are the first to argue that a link between public education and public pensions does or should exist. Pogue and Sgontz (1977) make this point in the context of a simple model of social security taxation. While they do not fully develop the dynamic implications of their argument, or bring it to the data, they stress that "the investment incentive provided by [pay-as-you-go payroll tax] financing is for *collective* investment by each generation in capital that will enhance the income of persons who will be working during the generation's years of retirement" (p. 163, italics in original). Richman and Stagner (1986) also argue, albeit even more informally, that the very existence of a pay-as-you-go pension system should generate an incentive for the older cohorts to invest in the younger ones. Further, a very large demographic, sociological, and anthropological literature has long argued that such intergenerational links (within the family, the clan, the village, or the entire society) are critical for understanding both fertility choices and parental investments in children. Caldwell (1978) and Nugent (1985) are recent references, while Neher (1971) is a very early economic paper in which fertility choices are linked to the parental desire to draw a pension when old.

In recent years, other authors have addressed a more general but closely related issue in the context of the overlapping generations model. That is: If current generations are selfish, why should they invest in assets that are valuable only to future generations? Symmetrically, what does lead the young generations to transfer resources to the old ones, who will not be around tomorrow? Kotlikoff, Persson and Svensson (1988) is an earlier reference: they cast the problem in terms of time-consistency of the optimal policy. The solution proposed involves a social contract which is "sold" by the old to the young generation in exchange for tax revenues. Boldrin (1992) and Boldrin and Rustichini (2000) analyse public education and public pensions, respectively. In the first case, education is publicly financed because it increases the productivity of future private physical capital, which provides the old generation with a channel to collect (part of) the return on their investment. In the second case, pensions are paid because they allow the working generation to act as a "monopolist" in the supply of savings, and therefore earn a higher total return on its investment. Subgame perfectness is used to show that an equilibrium with social security can be sustained. Conley (2001) and Rangel (2003) address the problem in a more general setting and reach the following general conclusion: Establish intergenerational arrangements such that future payoffs accruing to generations not born at the time the investment was made are transferred backward to the generation which made the investment. Rangel (2003) derives an interesting theory of "backward" and "forward" public goods on the basis of these premises. He uses game theoretical arguments, not dissimilar from those used in Boldrin and Montes (2001), to show that an equilibrium exists in which all generations play a trigger strategy guaranteeing that the appropriate amount of (backward) public goods is purchased. While Rangel's argument is developed in the context of a stationary exchange economy, it can be generalized to one with production and endogenous growth. Conley (2001) shows that when the public goods in question are durable and there is land, the Tiebout solution of providing the public goods locally achieves the efficient allocation. Finally, Bellettini and Berti Cerroni (1999) also use an overlapping generations model with production to argue that the existence of pay-as-you-go pensions which are financed by labour income taxation may not necessarily reduce growth. They do so by introducing public capital in the production function and using game theoretical arguments to show that, when pensions are financed by taxes on future labour income, there exists a subgame perfect equilibrium in which investment in the public good and economic growth are higher than otherwise. Finally, Cremer, Kessler and Pestieau (1992) also consider education and pensions as tools to alleviate inefficiency when altruism is absent. In their cases, though, investment decisions are taken by parents on behalf of their children, which leads to conditions for efficiency which are different from ours. In particular, in their analysis efficiency fails due to a lack of coordination between contiguous generations and not because of the missing credit markets, hence public education alone is enough to restore efficiency. Because they use an exchange economy, the issue associated to capital accumulation, growth, efficient allocation of savings, and optimal retirement portfolio, which are central to our analysis, are not considered.

While the positive predictions of our model may prove valuable to understand the historical origins of public education and public pensions, it is on the normative prescriptions that we wish to put our emphasis. Should the public education and the public pension systems be designed according to the rules presented here? We believe they should. Would this be practically feasible? We discuss three possible implementations which make use of the traditional tools of public policy: taxes, subsidies, transfers, and public debt. Our results are highly stylized and, exception made for the case in which lump-sum tools are available, we are not arguing that the complete market allocation can be perfectly replicated via the public education and public pension systems described here. Still, our theoretical analysis shows that, even in the presence of distortionary taxation, our prescriptions are likely to lead to an improved allocation, while our empirical analysis of the Spanish data (see Boldrin and Montes, 2001) shows that, mostly because of the ongoing demographic changes, the intergenerational flows implied by our criteria would be substantially different from those originated by the current system.

2. THE BASIC MODEL

2.1. Complete markets

Consider an overlapping generations economy in which agents live for three periods. Within each generation individuals are homogeneous, and each generation has a constant size of one. Physical capital, k_t, and human capital, h_t, are owned, respectively, by the old and the middle aged agents. The output of the homogeneous commodity is $y_t = F(h_t, k_t)$, where $F(h, k)$ is a constant returns to scale neoclassical production function. Young agents are born with an endowment h_t^y of basic knowledge, which is an input in the production of future human capital $h_{t+1} = h(d_t, h_t^y)$. With d_t we denote the physical resources invested in education. We assume that competitive markets exist in which young agents can borrow such resources. The function $h(d, h^y)$ is also a constant returns to scale neoclassical production function. During the second period of life, individuals work and carry out consumption-saving decisions. When old, they consume the total return on their savings. We assume agents draw utility from (c_t^m, c_{t+1}^o), denoting consumption when middle age and old, respectively. Neither consumption when young, nor leisure, nor the welfare of descendants affects lifetime utility. Adding such considerations would only increase the notational burden without contributing additional insights.

Let the homogeneous commodity be the numeraire. Output y_t is allocated to three purposes: aggregate consumption $(c_t = c_t^m + c_t^o)$, accumulation of physical capital (k_{t+1}), and investment in education (d_t). Human capital and physical capital are purchased by firms at competitive prices equal, respectively, to $w_t = F_1(h_t, k_t)$ and $1 + r_t = F_2(h_t, k_t)$ (subscripts of functions indicate partial derivatives). Aggregate saving finances investment in physical and human capital $(s_t = k_{t+1} + d_t)$, accruing a total return equal to $(1 + r_{t+1})s_t = R_{t+1}s_t$. Adding uncertainty in the returns from h_t and k_t would only strengthen our findings, as should be clear from the discussion at the end of this section.

The life-cycle optimization problem for an agent born in period $t - 1$ is

$$U_{t-1} = \max_{d_{t-1}, s_t} \{u(c_t^m) + \delta u(c_{t+1}^o)\} \tag{2.1}$$

subject to:

$$0 \le d_{t-1} \le \frac{w_t h_t}{R_t}$$
$$c_t^m + s_t + R_t d_{t-1} \le w_t h_t$$
$$c_{t+1}^o \le R_{t+1} s_t$$
$$h_t = h(d_{t-1}, h_{t-1}^y).$$

First order conditions simplify to:

$$u'[w_t h(d_{t-1}, h_{t-1}^y) - s_t - R_t d_{t-1}] = \delta R_{t+1} u'[s_t R_{t+1}] \tag{2.2a}$$

$$[w_t h_1(d_{t-1}, h_{t-1}^y) - R_t] = 0. \tag{2.2b}$$

The first condition is the usual equality between the interest factor and the marginal rate of substitution in consumption. The second equates the private return from investing in human capital to the cost of financing it via the credit market. A *Competitive Equilibrium* is defined by (2.2) and:

$$F(h_t, k_t) = c_t + s_t \tag{2.3a}$$

$$F_1(h_t, k_t) = w_t \tag{2.3b}$$

$$F_2(h_t, k_t) = R_t \tag{2.3c}$$

$$s_t = d_t + k_{t+1}. \tag{2.3d}$$

Given an exogenous sequence of human capital endowments for the young, $\{h_t^y\}_{t=0}^{\infty}$, one can solve the two blocks of equations (2.2) and (2.3) for (d_t, h_{t+1}, k_{t+1}), $t = 0, 1, \ldots$, to obtain a dynamical system $\Phi : (d_{t-1}, h_t, k_t) \mapsto (d_t, h_{t+1}, k_{t+1})$. Given initial conditions (d_{-1}, h_0, k_0), Φ induces the equilibrium path $\{(d_t, h_{t+1}, k_{t+1})\}_{t=0}^{\infty}$; given the latter all remaining prices and quantities can be determined.

In our setting, the equilibrium rental–wage ratio R/w is a decreasing function of the factor intensity ratio $x = k/h$; that is,

$$\frac{R}{w} = \frac{f'(x_t)}{f(x_t) - x_t f'(x_t)} = \frac{R(x_t)}{w(x_t)} = \omega(x_t)$$

where $f(x) = F(1, k/h)$. Without loss of generality, the algebra leading from (2.2) and (2.3) to Φ can be simplified by means of three technical assumptions.

Assumption 1. *The function $h : \Re_+^2 \mapsto \Re_+$ is smooth. The function $g : \Re_+^2 \mapsto \Re_+$ satisfying $h_1[g(x, h^y), h^y] - \omega(x) = 0$ exists, is well defined, and continuous.*

Assumption 2. *The function $u : \Re_+ \mapsto \Re_+$ is strictly increasing, strictly concave, and smooth. Given numbers $I \geq 0$, $R \geq 0$, the function $V(I - z, Rz) = u(I - z) + \delta u(Rz)$ is such that* $\arg\max_{0 \leq z \leq I} V(I - z, Rz) = S(R, I)$ *has the form $S(R, I) = s(R) \cdot I$, with $s(\cdot)$ monotonically increasing.*

Assumption 3. *For all $t = 0, 1, 2, \ldots$, the endowment h_t^y, satisfies $h_t^y = \mu h_t$, $\mu > 0$.*

Under these hypotheses, tedious but straightforward algebra shows that, given d_{t-1}, the two-dimensional implicit function problem

$$h_{t+1} - h[g(x_{t+1}, h_t), h_t] = 0$$
$$s[R(x_{t+1})][w(x_t)h_t - R(x_t)d_{t-1}] - k_{t+1} - g(x_{t+1}, h_t) = 0$$

has a well-defined solution:

$$h_{t+1} = \Phi^1(h_t, k_t) \tag{2.4a}$$
$$k_{t+1} = \Phi^2(h_t, k_t). \tag{2.4b}$$

Standard methods can be used to show that, given (h_t, k_t) and d_{t-1}, the equilibrium choice of (h_{t+1}, k_{t+1}) is unique and induces an efficient allocation of resources in period t. This amounts to *static efficiency*: In each period aggregate savings are allocated to equalize rates of return between the investments in physical and human capital. In the presence of uncertainty, this condition can be easily re-stated to take into account risk and correlation between assets' returns. Dynamic efficiency is subtler. It requires that, given (d_{-1}, h_0, k_0), there exists no feasible path $\{(\hat{k}_t, \hat{h}_t)\}_{t=0}^{\infty}$ which delivers more consumption than the competitive equilibrium during some periods without requiring less consumption during any other period. In our setting, one can use the characterization of dynamically efficient paths obtained by Cass (1972). To apply the original argument one must account for the possible unboundedness of consumption paths, which requires normalizing all variables by a factor growing at the balanced growth rate.[1] Under our assumptions, the technology set is a convex cone, and unbounded paths are feasible. They are an equilibrium if the utility function allows for enough intertemporal elasticity of substitution in

1. Technical details are available from the authors upon request.

consumption. In this case, the dynamical system (2.4) does not have any fixed point of the type

$$h^* = \Phi^1(h^*, k^*)$$
$$k^* = \Phi^2(h^*, k^*)$$

other than the origin, and equilibria converge to a (unique) balanced growth path characterized by a constant growth rate and a constant ratio $x^* = k^*/h^*$. We illustrate our results through a simple example.

Example. Let $u(c) = \log c$, $F(h, k) = A\cdot k^{\alpha} h^{1-\alpha}$, and $h(d, h^y) = B\cdot\lambda(h^y)d^{\beta}$, $\alpha \in (0, 1)$, $\beta \in (0, 1)$, $A \geq 1$, $B \geq 1$, with $\lambda : \Re_+ \mapsto \Re_+$ continuous and monotonically increasing. Manipulating the first-order conditions yields

$$s_t = \frac{\delta}{1+\delta}[w_t h_t - (1 + r_t)d_{t-1}]$$
$$d_{t-1} = \frac{\beta(1-\alpha)}{\alpha} k_t.$$

Setting $\frac{\beta(1-\alpha)}{\alpha} = \gamma$ and using the market-clearing condition for saving and investment gives

$$d_{t-1} = \frac{\gamma s_{t-1}}{1+\gamma}.$$

Hence,

$$k_{t+1} = A\eta[k_t^{\alpha} h_t^{1-\alpha}] \quad (2.5a)$$
$$h_{t+1} = B\lambda(h_t^y)(A\gamma\eta)^{\beta}[k_t^{\alpha} h_t^{1-\alpha}]^{\beta} \quad (2.5b)$$

where $0 < \eta = \frac{\delta}{1+\delta}\frac{(1-\alpha)(1-\beta)}{1+\gamma} < 1$. Let $h_t^y = h_t$ and $\lambda(h) = h^{1-\beta}$. Then (2.5) becomes

$$k_{t+1} = A\eta(k_t^{\alpha} h_t^{1-\alpha}) \quad (2.6a)$$
$$h_{t+1} = B(A\gamma\eta)^{\beta}(k_t^{\alpha\beta} h_t^{1-\alpha\beta}). \quad (2.6b)$$

The only rest point of (2.6) is the origin. The ray

$$x^* = \frac{k_t}{h_t} = \left[\frac{A\eta}{B(A\gamma\eta)^{\beta}}\right]^{\frac{1}{1-\alpha(1-\beta)}} \quad (2.7)$$

defines a balanced growth path. For all initial conditions $(h_0, k_0) \in \Re_+^2$, iteration of (2.6) leads (h_t, k_t) to the ray x^*.

Along the balanced growth path, the two stocks of capital expand (or contract) at the factor

$$1 + g^* = A\eta\left[\frac{B(A\gamma\eta)^{\beta}}{A\eta}\right]^{\frac{1-\alpha}{1-\alpha(1-\beta)}}$$

which is larger than one (*i.e.* there is unbounded growth) when

$$\eta > \frac{1}{A}\cdot\left[\frac{1}{B^{1/\beta}\gamma}\right]^{(1-\alpha)}.$$

A sufficient condition for the equilibrium path to be dynamically efficient is that the gross rate of return on capital be larger than or equal to one plus the growth rate of output. With linearly homogeneous production functions, the rate of return on capital is determined by the factor intensity ratio. Hence we need

$$(1 + g^*) < \alpha A(x^*)^{-(1-\alpha)}.$$

The latter reduces to $\alpha > \eta$, which is equivalent to

$$\frac{(1-\alpha)(1-\beta)}{\alpha+\beta(1-\alpha)} < \frac{1+\delta}{\delta}.$$

For reasonable values of α, β, and δ the latter is easily satisfied.

2.2. Equilibrium when credit markets are missing

In reality, credit markets financing educational investments are rare. The reasons for such a lack of privately provided credit are various and widely studied. (See, Becker, 1975 for a classical discussion, Kehoe and Levine, 2000 for a more recent one.) Lack of borrowing opportunities for the young generation implies that $d_t = 0$ for all t and $h_{t+1} = h(0, h_t^y)$. This makes the complete market allocation (CMA, from now on) unachievable and, by eliminating investment in human capital, leads the economy to an inefficient equilibrium. In fact, for the particular functional forms chosen in the example this leads the economy to a rather quick extinction; in general, the specific properties of the equilibrium without private education financing depend upon the assumptions one is willing to make about $h(0, h^y)$. This is not our concern here, as all such equilibria are inefficient in any case. Our interest lies, instead, with the CMA as a theoretical benchmark and with the class of intergenerational transfer policies that are capable of replicating it in these circumstances. We now turn to this issue.

3. INTRODUCING THE INTERGENERATIONAL STATE

When $d_t = 0$ in all periods condition (2.2b) is violated and $F_2(h_t, k_t) = R_t < w_t h_1(0, h_{t-1})$ holds. Profitable investment opportunities exist, which cannot be exploited. Too much is invested in the physical stock of capital, the k/h ratio is too high, and the rate of return on capital is too low with respect to the benchmark case. The allocation is inefficient: The young could increase their lifetime income by borrowing in order to accumulate human capital, and the middle aged could increase their retirement income by shifting some savings from k_t to d_{t-1}, but both are prevented from doing so. Apparently, such inefficiencies can be erased, and the CMA restored by a simple policy of taxing the middle aged an amount equal to d_t, to be spent in financing the education of the young. It turns out that, in general, this statement is not correct, and that a more sophisticated kind of public policy is required to fully restore CMA and efficiency. More precisely

Proposition 1. *If credit markets for investment in human capital are missing, a policy of taxing the middle aged a lump sum amount d_t to finance the education of the young implements a competitive equilibrium with the following properties:*

- (i) *the allocation it induces is different from the CMA;*
- (ii) *it may be inefficient, both statically and dynamically;*
- (iii) *when it achieves efficiency, it does so by making the initial generation worse off than under the CMA.*

To prove this, assume the initial conditions (k, h) are as in the CMA, and the policy described above is implemented. To lighten notation we discuss the case of a balanced growth path only, but the argument applies to the general case. Consider a middle aged person in period t, when the economy is growing at the factor $G = 1 + g$ and the return on capital is $R = 1 + r$. The policy requires a transfer of Gd from the middle aged to the young. Let the labour income

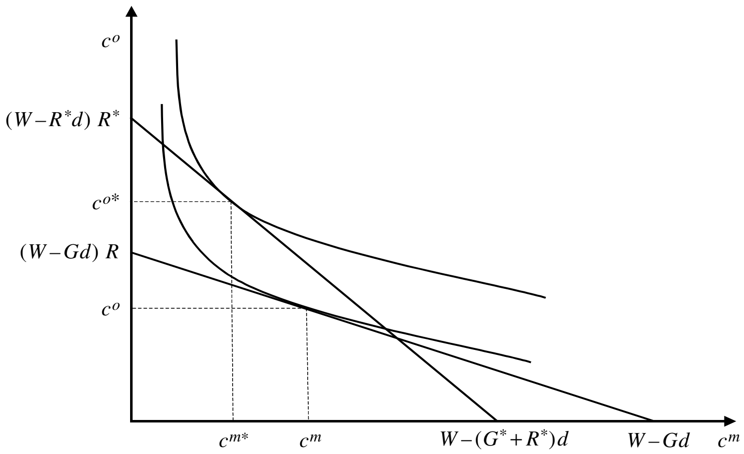

FIGURE 1

of the middle aged agent be W. The endowment of this agent in this and the next period, when old, is $Z=[(W-Gd),0]$, see Figure 1. The budget constraints read: $c^m+s=W-Gd$, for this period and $c^o=Rs$, for the next. The agent chooses $s\geq 0$, yielding a consumption pair (c^m, c^o) on the intertemporal budget line $c^o=R(W-Gd-c^m)$, also reported in Figure 1. To help the intuition, let the utility function be separable and logarithmic, with discount factor δ. Then we have

$$c^m=\frac{W-Gd}{1+\delta};\qquad c^o=R\delta\frac{W-Gd}{1+\delta};$$

and

$$s=\delta\frac{W-Gd}{1+\delta}.$$

Consider the same agent in a world with complete markets (recall that starred symbols refer to the CMA). She borrowed d and must pay back R^*d plus lend G^*d to the young; in exchange she will receive R^*G^*d next period. Her endowment is $Z^*=[(W-(G^*+R^*)d), R^*G^*d]\neq Z$. We will show that $R^*\geq R$ and $G^*\leq G$, with strict inequalities holding in general. Given (k,h), she picks s^*, yielding a consumption pair (c^{m*}, c^{o*}) on the intertemporal budget line $c^{o*}=R^*[W-(R^*+G^*)d-c^{m*}]+R^*G^*d$, $c^{m*}\leq W-(R^*+G^*)d$. This is also reported in Figure 1. *Query 1*

Again, in the case of logarithmic separable utility we have

$$c^{m*}=\frac{W-R^*d}{1+\delta};\qquad c^{o*}=R^*\delta\frac{W-R^*d}{1+\delta};$$

and

$$s^*=\delta\frac{W-R^*d}{1+\delta}-G^*d.$$

Here, as in the general case, $s^*\leq s$ holds. This implies that $R^*\geq R$ and $G^*\leq G$. This shows that the two allocations are different. It also shows when this policy fails to achieve efficiency. When returns on physical capital are decreasing, restriction (2.2b), cannot be met if $s>s^*$. Static efficiency is violated. This can be avoided by choosing d_t in such a way that (2.2b) holds in equilibrium. Given (k,h), this requires $d_t>d^*$. Decreasing returns then imply that $k_{t+1}=s_t$ would still exceed $k^*_{t+1}=s^*_t$ and $R<R^*$ would again hold. This has two

implications. First, the lower rate of return on capital may violate dynamic efficiency. Second, the first few generations will be worse off under this policy than under the CMA.

To understand the sources of this intergenerational redistribution, compare the lifetime utility of the first middle aged generation under the two arrangements (CMA and education financing) when the initial debt toward the old generation is zero. This assumption guarantees that the utility of the first old generation is the same in the two settings. In the CMA environment, the endowment of the first middle aged generation is $Z^* = (W - d^*, R^* d^*)$, while in the education financing environment we have $Z = (W - d, 0)$. As we have shown, $d^* \leq d$ holds and Z^* strictly dominates Z. Further, $s > s^*$ and so $R^* > R$. Hence the set of feasible (c^m, c^o) is strictly larger in the CMA than in the efficient education financing environment, which implies that the middle aged in period $t = 0$ are strictly worse off in the second environment. Depending on parameter values, this may be true for a (finite) number of generations after that. Consider now the general case in which the initial debt $d_{-1} > 0$. In this case, when d_{-1} is very large it is possible that the first old generation bears all the burden of the intergenerational transfer, while all other generations are better off under the education-financing-only policy. In any case, an intergenerational redistribution takes place as the education-financing-only policy always leads to “too much” investment in physical capital and “too little” consumption for the first few generations. This induces a higher growth rate and, therefore, may benefit generations alive far in the future, as they enter life with a higher initial endowment of k and h than otherwise. But this occurs at the cost of reducing the welfare of the initial generations.

The solution to this problem is: In each period, middle aged individuals must pay back their debt to the people of the old generation, who lent them in the first place. When this repayment is enforced the old generation collects the amount $R_{t+1}(k_{t+1} + d_t)$ as in the CMA, and the incentive to overinvest in k_{t+1} disappears. Notice that the portion $R_{t+1} d_t$ corresponds to an intergenerational transfer mediated by the government. Crucially, efficient education finance today and efficient pension payments tomorrow are tied together by a rate of return restriction.

3.1. *Publicly financed education and pay-as-you-go pensions*

Consider the following scheme. In each period t, two lump-sum taxes are levied on the middle aged generation, and the proceeds are used to finance, respectively, pensions for the old and education for the young. We assume a period-by-period balanced budget. Write

$$T_t^p = P_t, \tag{3.1}$$

for the pension scheme, and

$$T_t^e = E_t, \tag{3.2}$$

for the education plan. The budget constraints for the representative member of the generation born in period $t - 1$ become

$$0 \leq d_{t-1} \leq E_{t-1} \tag{3.3a}$$

$$c_t^m + s_t \leq w_t h_t - T_t^p - T_t^e \tag{3.3b}$$

$$c_{t+1}^o \leq R_{t+1} s_t + P_{t+1}. \tag{3.3c}$$

Comparison of equations (3.3) with the budget restrictions of problem (2.1) shows that, if the lump-sum amounts satisfy

$$E_t = d_t^*, \qquad P_t = d_{t-1}^* R_t^* \tag{3.4}$$

the competitive equilibrium under the new policy achieves the CMA. A benevolent planner can restore efficiency, improve long-run growth rates, and preserve intergenerational fairness by establishing publicly financed education *and* pay-as-you-go pensions simultaneously, and by linking the two flows of payments via the market interest rate.[2]

Efficiency properties aside, a Public Education and Public Pension scheme (PEPP) satisfying restrictions (3.1), (3.2), and (3.4) would also be actuarially fair in the following sense. The pension payment (contribution) that a typical citizen receives (pays) during the third (second) period of life corresponds to the capitalized value of the education taxes (transfers) the citizen contributed (received) during the second (first) period of life. These quantities are capitalized at the appropriate market rate of interest:

$$E_t R^*_{t+1} = T^p_{t+1} \tag{3.5a}$$

$$T^e_t R^*_{t+1} = P_{t+1}. \tag{3.5b}$$

In the applied literature on contribution-based social security systems, the issue of actuarial fairness between contributions paid and pensions received is an actively debated topic. Our model suggests that one should look for actuarial fairness somewhere else, that is, between contributions paid and the amount of public financing for education received on the one hand, and between taxes devoted to human capital accumulation and pension payments on the other. This observation is not irrelevant for the ongoing debate about the "sustainability" of public pension systems in the USA and Europe alike.

3.2. Distortionary taxation

In this subsection we ask if the CMA can be implemented as a competitive equilibrium with linear income taxes and period-by-period budget balance. In this model, the quantity E_{t-1} accruing to the young is effectively a lump sum. For given interest rate, so is the amount $T^p_t = R_t E_{t-1}$ to be repaid as a middle aged person; while the latter may be either mortagaged to be paid over an extended period of time, or reimbursed upfront, its net present value is given and, therefore, cannot possibly affect labour supply during the middle aged period. Distortionary effects may come from the income taxation needed to finance expenditure in education, and from the pension payments. We consider these issues next.

Taxing labour income distorts the borrowing/lending decisions of both young and middle aged individuals, making the CMA not implementable. Hence, at least in general, when lump-sum instruments are not available one cannot claim that the public education and public pension system we suggest would fully restore efficiency. Nevertheless, it seems reasonable to argue that the distortionary effects induced by the labour income tax cannot possibly be worse than those that current social security and income taxes already induce. Interestingly, taxing savings and subsidizing capital income may, at least in certain cases, implement the CMA.

Proposition 2. *For $t \geq 0$, let E_t be as in the CMA, and let $T^p_{t+1} = R_{t+1} E_t$. For $t \geq 0$, set τ_t so that s_t is at the CMA level. For $t \geq 1$, subsidize capital income $R_t s_{t-1}$ at the rate τ_{t-1}. For given initial conditions, this policy implements the CMA.*

Given (k_t, h_t), the choice of $E_t = E^*_t$ is obvious. Let us construct the sequence of taxes $\{\tau_t\}^\infty_{t=0}$ that implement the CMA. Use $\hat{x}$ and x^*, respectively, to distinguish variables in the

2. Introducing individual heterogeneity and income uncertainty complicates but does not alter these conclusions. See Boldrin and Montes, 2003*a* for details.

competitive equilibrium with taxes, and in the CMA. For all $t \geq 0$, set $\tau_t = E_t^*/s_t^*$. The budget constraints become

$$0 \leq \hat{d}_{t-1} \leq E_{t-1}^* \tag{3.6a}$$

$$\hat{c}_t^m + (1+\tau_t)\hat{s}_t \leq \hat{w}_t \hat{h}_t - T_t^p \tag{3.6b}$$

$$\hat{c}_{t+1}^o \leq \hat{R}_{t+1}(1+\tau_t)\hat{s}_t. \tag{3.6c}$$

The first-order condition determining $\hat{d}_{t-1}$ is identical to (2.2b), hence $\hat{d}_{t-1} = d_{t-1}^*$ holds. The condition determining $\hat{s}_t$ becomes

$$u'[\hat{w}_t \hat{h}_t - (1+\tau_t)\hat{s}_t - T_t^p](1+\tau_t) = \delta \hat{R}_{t+1} u'[(1+\tau_t)\hat{s}_t \hat{R}_{t+1}](1+\tau_t). \tag{3.7}$$

Simplifying and replacing T_t^p with $\hat{R}_t E_{t-1}^*$, and τ_t with E_t^*/s_t^* yields (2.2a), which has the desired solution $\hat{s}_t = k_{t+1}^*$. Further, $\hat{R}_{t+1}\tau_t\hat{s}_t = R_{t+1}^* d_t^*$, which corresponds to the CMA return from human capital assets.

For the simple economy considered in the Example, the choice of a constant $\tau_t = \gamma$ suffices to implement the CMA along any path. In general, a constant tax rate suffices along a balanced growth path when the production functions are linearly homogeneous. For production functions that are not linearly homogeneous or outside the balanced growth path, the tax rate cannot be constant because, in those circumstances, the composition d_t^*/k_{t+1}^* of the CMA investment portfolio is neither.

3.3. *Using debt*

Consider, finally, the case in which, instead of financing education via taxation, the government issues one-period, ear-marked debt in the amount E_t^* in each period. In the following period, the government pays back $R_{t+1}E_t^*$ to the debt holders. Such repayment is financed by a tax on the middle aged individuals, proportional to their past usage of public education financing. As mentioned above, the net present value of this tax is effectively a lump sum for the middle aged worker, independently of repaying the whole amount up front or through successive instalments. In principle, at least, the debt-repayment stage could be kept exempt from distortionary effects on labour supply. In practice, collection schemes and various minimum income provisions are likely to make the debt instrument also somewhat distortionary. What is important to notice, though, is that in this scheme the government effectively acts as a financial institution, issuing and managing the missing securities.

3.4. *Our model and the real world*

We view our analysis as essentially normative. When private competitive markets for financing education are not available, a properly designed intergenerational scheme may restore the efficient CMA as a competitive equilibrium. Our analysis also shows that the distance between actual and efficient allocations, at least along this specific dimension, can be measured by looking at the difference between some implicit rates of return, which can be measured in the data, and the market rate of return.

Proposition 3. *If the set of intergenerational transfers induced by the public education and the public pension systems supports the CMA, the following should be observed. For a given generation, the implicit rate of return i_t which, along the life cycle, equalizes the discounted values of education services received and social security contributions paid, is equal to the market rate of interest r_t. Similarly, the implicit rate of return π_t that, along the life cycle,* *equalizes the discounted values of education taxes paid and pension payments received, is also equal to the market rate of interest* r_t.

As reality is seldom, if ever, fully efficient, it becomes relevant to ask how much "off the mark" current intergenerational arrangements are. The quantities $|\pi_t - r_t|$ and $|i_t - r_t|$ are reasonable ways of measuring such distance. Should reality turn out to be not far from what we have shown to be the efficient allocation, it would become an interesting topic of research to ask how existing political mechanisms implement allocations that closely satisfy the Pareto criterion. Should, instead, reality turn out to be far from the efficient allocation, then it becomes relevant to ask how one should proceed to bring it closer.

These considerations lead us to entertain, albeit briefly, a positive reading of our model. In the real world benevolent planners are probably harder to come across than credit instruments for financing education. *A priori*, there are very few reasons to expect that existing public education and pension systems should strive to replicate the CMA and achieve the efficiency gains we have outlined through our model. As a matter of fact, in none of the countries we are aware of is the welfare state legislation explicitly organized around the principles advocated in this paper. In general, social security contributions are levied as a percentage of labour income and bear no clear relation to the previous use of public education. Pension benefits received are related, in one form or another, to past social security contributions but never to some measure of lifetime contributions to aggregate human capital accumulation. Still, there are intuitive reasons to believe that intergenerational transfers that are either grossly inefficient or openly unfair (in the sense that some generations collect rates of return systematically higher than those of other generations) would be subject to strong public pressure to be either dismantled or improved upon. This is the intuition set forth by Becker and Murphy (1988) and which is captured in our model by conditions (3.5). In particular, as those equations show, both fairness and replication of the CMA are summarized by a simple present-value calculation that uses the market rate of return as a yardstick.

In a recursive environment in which the middle aged generation decides whether and how to implement an intergenerational transfer system of the kind studied here, an equilibrium satisfying (3.5) may in fact arise. In earlier working paper versions of this article, we presented a dynamic game of generational voting, along the lines of Boldrin and Rustichini (2000), which possesses a subgame perfect equilibrium implementing the CMA. We refer the interested reader to Boldrin and Montes (2003*b*) for this result, a discussion of the circumstances under which the political equilibrium implementing the CMA is the unique subgame perfect and, finally, for extensions to other notions of recursive equilibrium, and to more general OLG environments. As already mentioned, results along the same lines have been derived independently by Rangel (2003).

We have also used micro and macro data to estimate the values of i and π faced by Spanish citizens during the 1985–1995 period. A detailed description of the data set used, estimation procedure, results, and simulations can be found in the working paper version of this article. It suffices to report here that our point estimate of the implicit annual rate of return on education investment is $\pi = 4 \cdot 238\%$ while, depending on accounting criteria, our point estimate of the annual rate charged by the government on its educational lending goes from $\underline{i} = 3 \cdot 6307\%$ to $\bar{i} = 4 \cdot 2601\%$. Various considerations suggest that the latter is a much better estimate of the real cost to private citizens of receiving public education services, given the current Spanish institutional setting and legislation. An implication of these findings is that, if it were not for the dramatic demographic transition the country is currently undergoing, the set of intergenerational transfers that the Spanish public pension and education systems induce would not be very far from what the complete market allocation might be.

4. CONCLUSIONS

We have studied a three-period overlapping generations model with production and accumulation of physical and human capital. When the young generation cannot borrow to finance investment in human capital, the competitive equilibrium outcome does not satisfy either static or dynamic efficiency, and the aggregate growth rate of output and consumption is lower than under the complete market allocation. We show that a simple intergenerational transfer agreement could eliminate this problem and induce an efficient allocation.

The intergenerational transfer agreement we study is inspired by the argument advanced in Becker and Murphy (1988). Accordingly, we interpret public financing for education as a loan from the middle aged to the young generation. The latter uses this loan to finance its accumulation of human capital. Symmetrically, the pay-as-you-go public pension system can be seen as a way for the former borrowers to repay the capitalized value of their education debt to the previous generation. In this interpretation, the two institutions of the welfare state, public education and public pensions, support each other and achieve a more efficient allocation of resources over time.

There are important normative implications of this analysis. Our model suggests that utilization of either public or publicly financed education should be treated as accumulation of debt toward the older generations. Such debt, capitalized at the market rate of interest, should be paid back, during one's working life, by means of a tax levied upon labour income. Repayment of the education debt can be achieved by means of a voluntary mortgage plan or by means of a compulsory tax. Either choice has some obvious incentive and redistributive implications, which are, nevertheless, not dissimilar from those faced by current arrangements for financing public education. On the side of retirement pensions, the model requires earmarking some tax (paid by individuals) as a source of public financing of education and to capitalize at the market rate of interest the amounts paid by each single citizen. The capital so accumulated should then be paid out, in the form of annuities, to the same citizen once retirement age is reached. These are our main theoretical and normative findings. They suggest that public education financing and a properly redesigned public pension system could be useful tools to enhance economic efficiency and long run welfare. While our abstract analysis leads to the conclusion that full efficiency can be restored with lump-sum tools, it is clear that, in practice, the use of distortionary taxes and transfers will make full efficiency not achievable. It is therefore an empirical question to evaluate the extent to which a scheme of the kind described here may improve over the existing one. The theoretical analysis suggests it should.

While a benevolent planner could easily implement such a system of lump-sum taxes and transfers, it is not obvious that a benevolent planner is behind the design of modern welfare state institutions. Hence, it is worth investigating if existing systems are or not far away from the quantitative prescriptions of our normative model. We do so by computing the "borrowing" and "lending" rates implicit in the Spanish public education and public pension systems. We use both microeconomic and aggregate data for 1990–1991. The model predicts that, at the CMA allocation, the borrowing and lending rates should equal each other and be equal, in turn, to the rate of return on capital. For the baseline case, our point estimates of borrowing and lending rates are relatively close to 4·0%, which corresponds to the risk-free real rate of return on Spanish Treasury bonds during the last 15 years or so. This optimistic finding, though, is based on the assumptions of demographic and policy stationarity. More work needs to be done to assess the full impact of such changes on intergenerational transfers and economic efficiency, as well as to understand the mechanisms through which our political institutions handle the economic consequences of demographic change. ***Query 2***

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Page 8

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Page 13

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Page 14

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