

# Mortality, Fertility and Saving in a Malthusian Economy\*

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# 1 Introduction

In this paper, we develop and analyze a simple model of fertility choice by utility-maximizing households. Following the work of Becker and Barro (1988) and Barro and Becker (1989),<sup>1</sup> we base our model on an explicit notion of intergenerational external effects. In contrast to the Barro and Becker model, however, we assume that the external effects run from children to parents. That is, parents' old-age consumption directly enters the utility function of the children, whereas in the Barro and Becker model, utility of children enters directly into the utility function of the parent. This gives rise to a fundamentally different reason for bearing children from what is present in the Barro and Becker model: parents expect to be cared for, at least partially, by their children in their old-age when their labor productivity is low. Thus, children are an investment in old-age consumption from the parent's point of view.

Moreover, in our model, parents understand that the amount of support they can expect from their children in old-age is itself dependent on the number of surviving children. We consider two alternative formulations of the decision a child has in providing old-age care for his parents. One is cooperative among the siblings; the other is noncooperative. We find that other things being equal, cooperation among the children gives rise to a substantially higher equilibrium level of support for parents, and hence leads to a much higher fertility choice. The difference in behavior is so substantial that, in reasonably parameterized versions of the model (discussed in Section 5), the non cooperative solution induces a level of fertility which does not even guarantee a constant population, whereas the cooperative behavior induces, in the same circumstances, levels of population growth that are high by historical standards. We discuss the empirical and positive relevance of these findings in Section 4.

An abundant demographic and anthropologic literature studies the direction of intergenerational family transfers, the motives to which they can be attributed, and the institutional and cultural arrangements supporting their

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<sup>1</sup> This approach dates back to Becker (1960). Razin and Ben-Zion (1975) contains an early dynamic formulation of the dynastic model of fertility choice.

existence. Caldwell (1978, 1982) makes an important contribution to this literature. He proposes a theory of the fertility transition based on the idea that transfers from children to parents are the reason behind high fertility. Changes in the social fabric that reduce or eliminate such transfers within the family bring about a reduction in fertility. In fact, Caldwell advances the hypothesis that the third stage of the fertility transition, which we discuss below, can be attributed mostly to such changes in socioeconomic norms within the familial unit. Willis (1982) provides an interesting discussion of Caldwell's theory and analyzes a stylized model of decision making in the settings envisioned in Caldwell's work. In related empirical work, Lillard and Willis (1997) find little support for the parental altruism motivation for child-bearing in Malaysian data on intergenerational transfers. They conclude, rather, that these transfers are consistent with either the investment point of view (which we take here) or the view that children are repaying parents for implicit loans made when the children were young (which is considered in Boldrin and Montes (1997)). Similar empirical findings are reported in Jensen (1990), while an even earlier assessment of the strong empirical evidence supporting the "old-age security" hypothesis can be found in Nugent (1985).

Our approach to modeling fertility choice is roughly consistent with Caldwell's hypothesis, that is that children are investment goods from the parents' viewpoint and that the desired number of children depends on, among other things, how much children transfer to old parents in relation to the cost of rearing them to adult life. We formalize the "changes in social norms" by comparing cooperative to noncooperative behavior. We also study the interactions between land/capital accumulation, infant mortality, and fertility choices. These themes are absent in the earlier theories.

One of the key features of our model is its comparative statics with respect to infant mortality. In the Barro and Becker model, a reduction in the infant mortality rate both decreases the cost of creating surviving children and increases the expected benefits since more children survive to consume. (See Barro and Becker (1989) and Fernandez-Villaverde (2001).) Because of these effects, reductions in the infant mortality rate give rise to increased fertility

in the standard formulation of this class of models.<sup>2</sup> As we argue in Section 2, this prediction appears to be strongly counterfactual. In our model, we the opposite prediction holds, for all configurations of the parameters. Since, from a parent’s perspective, children are simply an investment in their own future consumption, decreases in infant mortality decrease fertility.

The dynamics of the response to changes in infant mortality rates are also interesting. Fertility falls along the path of adjustment when infant mortality is steadily decreased, but this reduction occurs with a lag and is less than proportional. Thus, along the transition path, a decrease in infant mortality brings about both a decrease in fertility and an increase in the growth rate of the population. As we argue below, this pattern corresponds to the fundamental stylized fact of the second stage of what is defined as the demographic transition, which we discuss in Section 2.

The model also has interesting implications for fertility from an increasing access to financial markets and an increase in social security pensions. Both of these modifications to the model provide channels through which old people obtain income independently from the support of their children. In the cooperative case, increases in independent old-age income are offset by decreased support from children, leaving parents roughly neutral with respect to fertility. In the noncooperative case, increases in independent income are more than offset by reductions in support, and hence, fertility falls.

These properties of the model give rise to interesting possibilities for understanding some features of the fertility data. First, there is a high and positive correlation between infant mortality rates and total fertility rates in cross-sectional observations. (Care must be taken here in interpreting causality of course.) Second, the historical timing of the reduction in infant mortality in almost all countries gives rise to a predicted reduction in fertility in the model that is in some ways similar to that seen in the demographic transition. Third, the much lower levels of fertility induced, *ceteris paribus*, by the noncooperative model point to an important regularity across coun-

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<sup>2</sup> A more detailed and quantitative discussion of this issue is postponed until Section 5. It should be noted that Sah (1991) contains a qualitative analysis of a simplified, static, stochastic, discrete-choice version of the “utility from children” model, in which reductions in infant mortality do bring about reductions in fertility.

tries and historical periods if we hypothesize that urbanization is related to a reduction in cooperation among siblings. That is, urbanization and the dissolution of the dynastic family are accompanied by a rapid and dramatic drop in fertility rates, even in the absence of changes either in income per capita or in other relevant economic determinants of fertility. This is reminiscent of the earlier informal discussions in the demographic and anthropological literature we cited above. Finally, the properties of the model with respect to changes in social security are suggestive of the recent observed reductions in fertility across many developed countries.

We speculate that the last two properties of the model may also help explain the third phase of the demographic transition in which, in the face of low and basically constant rates of child mortality, fertility continues to drop, leading to a halt (or even a reverse) in the process of population growth. We do not pursue this line of investigation here, though. The potential of the model to be properly calibrated to historical data to provide a full, quantitative account of the main features of the historical process of demographic transition is therefore left for future research.

The demographic literature on both the old-age support aspect of child-bearing and the effects of infant mortality on fertility is extremely large and cannot be properly discussed here; Kirk (1996) and Preston (1996) are good starting points. On the economic side, there is a small but significant literature on the role played by these two aspects in the onset of the demographic transition. Surprisingly, at least from our vantage point, seldom, if ever, can one find infant mortality and old-age support modeled together as the driving forces behind long-run demographic movements until the dawn of the XX century. A large portion of this literature is set in a partial equilibrium framework and/or does not make any attempt to endogenize the size of the transfer from middle-age people to old parents and its linkage with saving behavior, arable land expansion, and capital accumulation. Those contributions which are closer to ours are discussed in detail in the next section.

Section 2 contains summaries of the relevant data and of previous theoretical work in this area. Sections 3 and 4 contain the specifications and development of the models that we analyze. Section 5 contains some simple and purely qualitative computational results for the basic model. Finally,

section 6 concludes and directions for future work.

## 2 Empirical Evidence and Earlier Literature

### 2.1 Summary of Historical Evidence

Several historical regularities in the data on population dynamics are well established. These include the historical response of fertility to the switch from hunter-gatherer societies to agricultural ones, the short-term response of fertility to sudden changes in the ratio of population to available and productive land (as determined, for example, by plagues, famines, and wars), and the demographic transition that began in Europe in the 18th and 19th centuries.

The switch from a hunter-gatherer to agricultural society is equivalent to the transition from a technology in which the amount of land controlled by a family is essentially fixed to one in which the amount of land controlled by a family is incremented through investment both in land and in children. The availability of additional land was accompanied by an increase in fertility (from a rate of about 5 to around 7 or 8 according to most archaeological evidence) and a small but visible reduction in overall standards of living. All along recorded history, the short-term responses to temporary changes in the ratio of population to arable land were to counter these changes with movements in fertility. Thus, plagues and famines were followed by bursts of fertility of approximately the same size. A similar pattern is observed after significant military conflicts. These facts are well documented in the demographics literature (for example in Livi-Bacci (1989)), and they should be treated as basic consistency requirements for any dynamic model of population behavior.

Demographic transition, as used by demographers, refers to the large-scale change from a state of high fertility and high mortality to a state of low fertility and low mortality that has taken place in many countries. These transitions began in the early part of the XVIII century in a subset of European countries and in China (see, for example, Livi-Bacci (1989) and Chesnais (1992).) Briefly, this was a period of rapid change marked

by a drastic improvement in public health followed by a quick reduction in mortality rates. The initial equilibrium is one of a slowly growing population with very high mortality and fertility rates. Mortality rates are not only high, but also extremely volatile, with spikes occurring more or less in correspondence with wars, famines, and epidemics. As mentioned earlier, fertility rates respond to sudden increases in mortality, rapidly bringing the population level back to where it was before, to resume then a pattern of slow secular population growth. When a long run reduction in mortality sets in, it is invariably accompanied by a rapid growth in population. In most countries, this reduction in mortality was followed, after a lag of from 20 to 60 years, by an equally large drop in fertility rates. Demographers (for example Chesnais (1992) and Kirk (1996)) have come to identify three phases of the demographic transition: (I) in the initial stage both mortality and fertility are very high, population growth is very low (on the order of half of a percentage point or less per year); (II) in the intermediate stage, both mortality and fertility are dropping but the first drops more than the second, at least initially; hence, population growth increases, reaching levels of 2 or even 3 percent per year in some cases; and (III) in the final phase, mortality reaches a lower bound together with fertility, and population stops growing and sometimes declines, like in many Western countries during the second half of the XX century. Quantitatively, this transition was very large. Before the transition began, both crude birth and crude death rates were in the range of 30 to 40 per 1000 per year. By the time the transition was over, these rates were on the order of 10 to 15.

In England, the most widely studied case, the reduction in mortality was followed by a short period of increased fertility, with a reduction following about a century later. A similar pattern was observed in France, although the length of the period of increased fertility was shorter and later. In most countries in Europe, no such increase in the fertility rate occurred. Rather, the reduction in the crude death rate was initially met with little or no response on the part of fertility. This caused the birth and death rates to spread apart, temporarily giving rise to a rapid increase in population before the reduction in fertility occurred. As we argue below, the drop in fertility follows, in almost every country, a drop in the mortality of infants

and children. Due to the availability of modern health techniques, the drop in mortality rates has been much faster in the demographic transitions of the second half of the XX century than in those of the previous two centuries. Even in these more recent transitions, however, the most common pattern has been one of constant fertility for one or two generations. This has brought about dramatic increases in the total population of the developing countries that have undergone the demographic transition during the second half of the XX century.

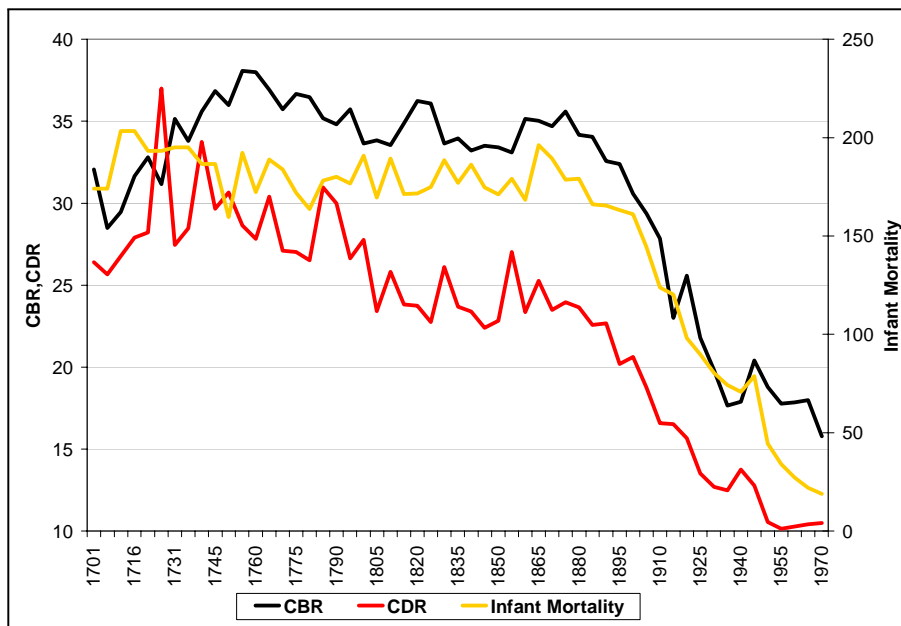


Figure 1: The European Demographic Transition

Figure 1 shows the typical pattern of crude birth and death rates that occurs in a demographic transition. This pattern was constructed as an average of the actual data from 11 European countries<sup>3</sup> over the period from

<sup>3</sup> These countries are France and the United Kingdom since 1740; Finland, Norway, and Sweden since 1755; Spain since 1797; Denmark since 1800; and Belgium, Germany, Italy, and the Netherlands since 1816.



1740 to 1991. As such, it mimics the overall pattern of each of the transitions while being less sensitive to individual country variations in the patterns of births and deaths. We have also superimposed the time series of average Infant Mortality Rates (IMR) on this figure as well. As can be seen, this follows a similar pattern, but the timing is different and should be noted. In particular, the drop in IMR is after that of the death rate, but precedes or is almost contemporary to that of the birth rate. In fact, the figure suggests that overall birth rates were roughly constant for a century or more while crude death rates were dropping and that fertility starts to decrease at roughly the same time that IMR's do. This observation is the crucial motivating factor behind the model we develop in Section 3, which concentrates on the impact of infant mortality on fertility and abstracts from changes in the total death rate. This behavior of the infant mortality rate and crude death and birth rates gives rise to the pattern of population shown in Figure 2a, which reports the total population for the European countries included in Figure 1.

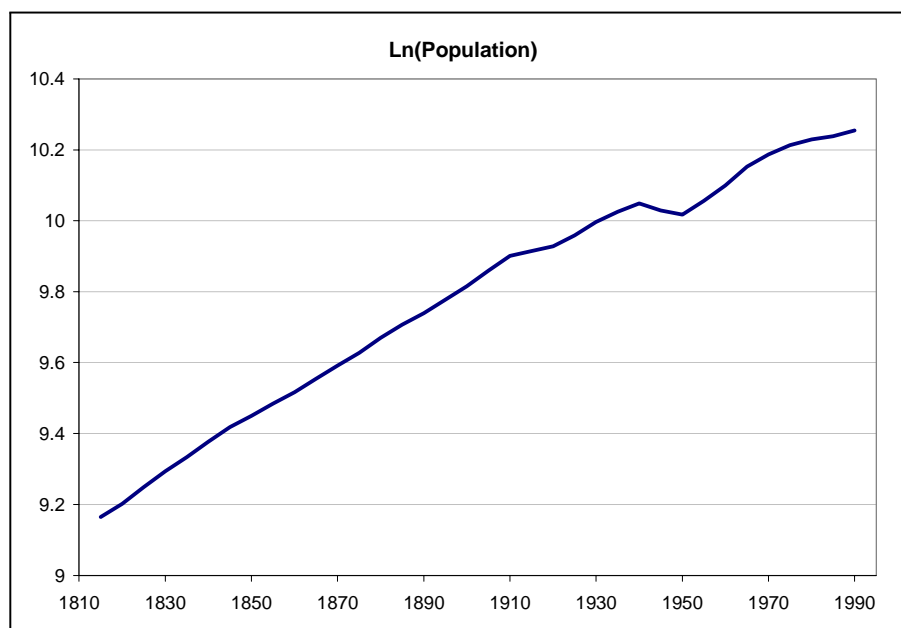


Figure 2a: The Population of Europe

Notice also a second important characteristic of the demographic data, which we ignore in our subsequent analysis for the sake of simplicity. The variability of the death rates, both total and infant, is much higher in the earlier than in the later period. Extending the time series backward to the XVII and XVI centuries would make the drop in volatility of death rates even more striking. Clearly, the variance of children's rates of survival must be a powerful factor behind the fertility choice of parents; an increase in the expected survival rate and a simultaneous reduction in its variance converge in pushing fertility rates down for a given expected value of desired children. Bringing uncertainty in death rates back into our model would therefore strengthen our conclusions.

Several explanations have been proposed for the demographic transition. The most widely accepted, at least among economists, argues that the increase in income that came with the Industrial Revolution jump-started the whole process. In the paradigmatic case of England in 1750-1900, the more than doubling of the population came together with the onset of the Industrial Revolution. This fact has led many economists (for example, Becker, Murphy, and Tamura (1990), Ehrlich and Lui (1991), Galor and Weil (1998), Lucas (1998), and Hansen and Prescott (1999)) to establish a causal link between the adoption of modern production technologies, the sustained increase in living standards they brought about, and the demographic transition.

While we cannot dismiss such a link, we believe its existence and direction are far from obvious. The reduction in mortality rates took place in many countries around the world in roughly the same period it took place in England. (See, for example, Livi-Bacci (1989).) An example of a country in which mortality drops and population growth takes off is China during the XVIII century. No industrial revolution occurred there during this time period. Demographic transitions have been occurring since then in almost every country around the globe with little if any correlation with the spread of the industrial mode of production. For example, Sub-Saharan countries are now in the middle of their demographic transition, with infant mortality and fertility rates dropping despite very little economic development. Another startling example is Cuba, which, with a stagnating income per capita that is orders of magnitude smaller than the one in the United States, ap-

pears to have reached crude and infant mortality rates and fertility rates that are indistinguishable from those in the USA. In these and other examples, income per capita progresses little if at all; the reduction in mortality rates is accompanied by a reduction in fertility, however.

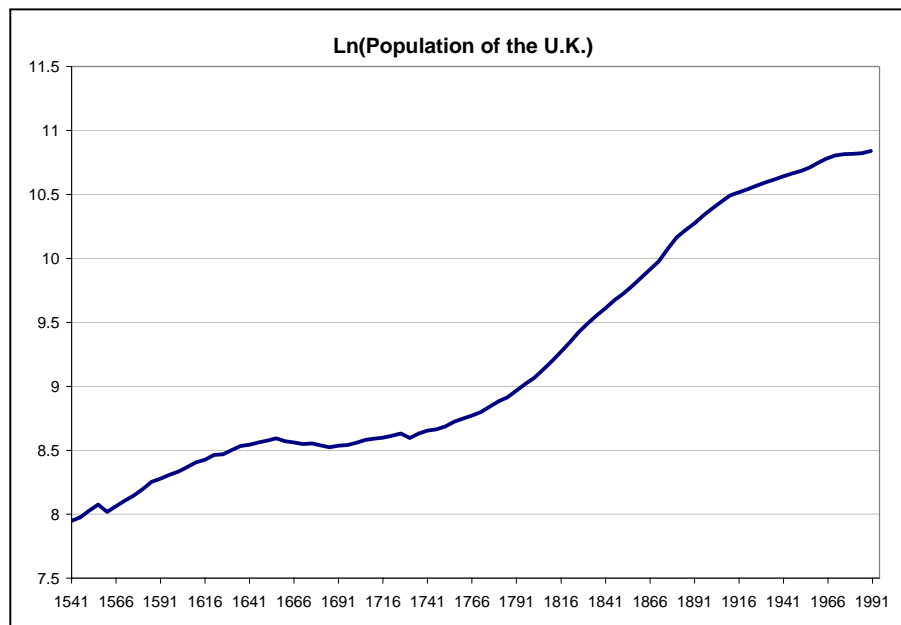


Figure 2b: The Population of the U.K.

Further evidence questioning the link between the income expansion generated by the industrial revolution and the onset of the demographic transition comes from the northern European countries, where the impact of the industrial revolution was felt earlier than in the rest of the world, but fertility rates typically declined much later, well after per capita incomes had started to grow. This evidence leaves open the question of whether or not there is some causal link between these two transitions. For example, although most researchers date the beginning of the increase in Gross Domestic Product (GDP) per capita in England sometime in the late 1700s, the real reduction in fertility does not begin to occur until somewhere between 1820 and

1890. As noted, the increase in fertility rates that began in England sometime around 1700 and lasted until 1820 makes England a particularly hard case to date. A more prototypical country might be Denmark (or, for that matter, any of the Scandinavian countries), where the reduction in fertility seems to begin around 1880 or 1890, well after the industrial revolution began. (Denmark is more prototypical because there is no sustained increase in fertility before the demographic transition began.) Sweden and the Netherlands follow a similar pattern with the reduction in fertility first starting in about 1860 to 1870. In all these cases, though, the strong correlation between mortality rates, especially infant mortality rates, and fertility is replicated. Further, the historical and demographic literature (for example, Livi-Bacci (1990)) has convincingly documented that a causal relation between income level, nutrition, and mortality seems to be present only in those extreme circumstances in which low nutrition leads to famine and massive mortality. For all intermediate ranges, covering the average nutritional levels which characterized European countries during the last millennium, the causality running from income and nutrition to mortality (and from the latter to fertility) appears dubious at best. To put it plainly: historical research seems to reject the idea that a pure increase in income levels and living standards would lead, or has historically led, to a reduction in fertility rates.

More important, this view has caused difficulties for quantitative researchers trying to develop models based on the hypothesis that the industrial revolution is at the root of the demographic transition. (See Doepke (2000) and Fernandez-Villaverde (2001).) In short, the basic idea is to build a model in which an increase in income generates a reduction in fertility. There are three problems. First, as noted, any model that does this faces difficulty because of the timing. Second, as income has continued to grow in industrial countries, fertility has not declined proportionally. Third, fertility rates have decreased dramatically in many countries where income per capita has increased only slightly or not at all.

What is uncontroversial is the set of quantitative facts describing the demographic transition, including the variable delay between mortality and fertility drops. A reduction in mortality rates is followed, perhaps with a lag to be explained, by a reduction in fertility. After a transition, fertility and

mortality rates converge to a new, much lower plateau, with almost zero or even negative natural growth of the population.<sup>4</sup>

The other key variable of interest from our point of view is the time series behavior of infant mortality rates. Infant deaths also went through a dramatic transition over this period, falling from levels of around 200 deaths per 1,000 in most countries in 1800 (and as high as 300 in, for example, Germany) to less than 20 in modern times, to reach a level below 10 per 1,000 in the most advanced countries during the 1990s. The bulk of this reduction occurred between 1880 and 1930 in most countries. For example, in England, the infant mortality rate was 150 per 1,000 in 1890 and 50 in 1930. There has been a more gradual, but continuing, decline since that time. Since the timing of this drop coincides with the reduction in fertility in many countries, we take this as an interesting potential explanation.

The cross-sectional evidence on these data is also of interest. There is a strong positive correlation between total fertility rates at the country level and the infant mortality rate in that country. This is evidenced in Figure 3, which shows cross-sectional data on IMRs and the Total Fertility Rates (TFR) in a collection of countries in 1997. As can be seen from the figure, there is a strong positive relationship between the TFR of a country in a year and the IMR of that country in the same year. However, care must be used in interpreting these data. IMR and income per capita are also highly correlated. (The correlation coefficient for 1997 is  $-0.814$ .) Because of this, it is natural to question whether or not this relationship just represents a spurious relationship, with income being the basic causal variable. Regression results offer some insight, with the effect of IMR on TFR being more statistically significant than log income.

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<sup>4</sup> Schultz (1969) is an earlier study pointing out the strong correlation displayed by the data. Since then, both micro and macro evidence has been mounting. For a partially dissenting view see van de Walle (1986).

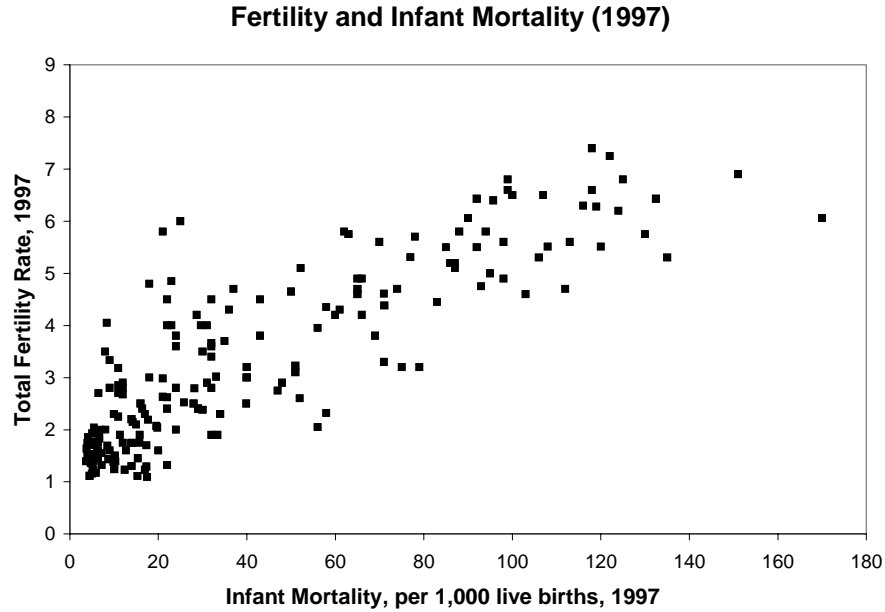


Figure 3: Cross Section IMR and TFR, 1997

For the 1997 data, we have

$$TFR_{97} = 3.24 + 0.0356IMR_{97} - 0.386 \log_{10} GDP_{97},$$

with an adjusted  $R^2$  of 0.763. The t-statistics for  $IMR_{97}$  and  $\log_{10} GDP_{97}$  are, respectively, 11.7 and  $-1.57$ . While the p-value associated to the  $IMR$  variable is practically zero, the one for per capita GDP ( $GDP_{97}$ ) is 12 percent. Moreover, this relationship seems to be remarkably stable over time. For the same data in 1962, there is a non linearity in the relationship, with what seems to be a biological maximum TFR of 7.5 or 8.0 at IMRs above 150 per 1,000 per year. Below this level, the estimated slope is 0.0387, virtually identical to the estimate from the 1997 data.<sup>5</sup>

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<sup>5</sup> Ehrlich and Lui (1991) have also investigated this issue using a cross section of data from various countries for the period 1960-85. They find that a decrease in the mortality rate of the younger group (ages 0 to 25) has a significant negative effect on fertility, while measures of longevity for the old group have no impact.

So far, all countries that have undergone these demographic transitions were essentially agricultural, at least when the transition started. The key feature of our modeling effort that differentiates it from previous work in a similar vein is the view that in poor, agricultural societies, the impetus for having children in the first place is as an investment rather than as consumption. That is, children are borne out of a need to man the farm when the parents grow older. In this arrangement, parents provide nurture to children when they are young, feed and clothe them, and provide them with land to work when they are older. In exchange for this, parents are cared for when they are old. This is a better arrangement than just selling the land on the market when parents are old, because, as argued by Rosenzweig and Wolpin (1985), children have learned to work the parents' land better than people from outside the family. Although one can imagine many mechanisms for implementing such an arrangement, (see Guner (1998) for a recent study), in this first set of models, we concentrate on a particularly simple one. This is an external effect running from parents to children. That is, children care about the well-being of their parents. This is in stark contrast to much of the previous literature in this area, where it is assumed that parents care about their children's well-being, but not vice versa. (Becker (1960) and Becker and Barro (1988) are the basic references.) This fundamental difference in motivations and modeling gives rise to some interesting and important differences in results and is at the heart of our research strategy.

## 2.2 Summary of Earlier Literature

The standard model in the economic literature is presented in two papers by Barro and Becker (1988) and (1989). In the general equilibrium version of the model, they assume that there is exogenous, labor-augmenting technological change and that the cost of child rearing is made up of two components: one part consumption goods, one part time. They assume that the motivation for bearing children in the first place is altruism on the part of parents. That is, parents get utility both from their own consumption and from the utility of their children. Barro and Becker analyze the balanced growth path of the model along with off-steady state dynamics. The character of this balanced

growth path is that fertility is constant so that population grows at a steady rate while income per capita grows. This is important from the point of view of the explanations for the demographic transition outlined above. That is, although income is growing in this model, fertility does not decline. For this reason, this model, as it is, does not provide a foundation for the story outlined above in which income growth (that is, the onset of the industrial revolution) provides the basis for the demographic transition.

Barro and Becker also analyze some simple comparative statics exercises across steady states. In particular, they look at the effect of a reduction in the costs of child rearing on fertility. In the general equilibrium version of the model, they show that across balanced growth paths, fertility increases when the cost of rearing children falls, because of the effect of that cost on interest rates. That is, since interest rates rise when the cost of rearing children falls, it pays to invest more in the future through a larger family.

One of the interesting and relevant interpretations of this comparative statics exercise is a change in the infant mortality rate. That is, if we reinterpret the cost of rearing children as a cost of producing a surviving child, then a reduction in the cost of rearing children is like a reduction in the infant mortality rate. Barro and Becker argue that therefore, a reduction in the IMR would give rise to an increase in the number of surviving children. This could be consistent with a reduction in the fertility rate as long as fertility did not fall as much as the probability of death.

Fernandez-Villaverde (2001) brings a quantitative analysis into the discussion by studying a calibrated version of the Barro and Becker model. He conducts three quantitative experiments with the model: an increase in productivity, a reduction in the infant mortality rate, and a fall in the relative price of capital. As expected due to Barro and Becker's balanced growth path results, the first experiment does not give rise to a demographic transition. Reducing the relative price of capital while holding the infant mortality rate fixed does give rise to reduced fertility and population growth, which in some ways is similar to the historical experience, but reducing infant mortality while holding the relative price of capital fixed moves both fertility and population growth rates in the opposite direction. He does not consider the experiment of changing both rates simultaneously. It is difficult to tell



which of these effects is larger and hence what the overall prediction of the model would be. However, one thing that unambiguously does come out of the model is that even a small reduction in IMRs increases both fertility and population growth rates. Further, the endogenous increase in fertility is quantitatively large for the drop in IMR considered by Fernandez-Villaverde, at least from a historical point of view. Finally, the model cannot provide any explanation of why fertility drops to such a low level that population growth comes to a full halt in the second half of the last century.

The paper by Doepke (2001) is in a similar vein. He analyzes a quantitative version of the Barro and Becker model and considers the effects of child labor laws and compulsory, subsidized education. He finds that each of these policy experiments has the potential to have big effects on equilibrium fertility and population growth rates. The key thing in the model that is driving the fertility decision is the relative cost of producing a new skilled child vs. a new unskilled child. It is through this avenue that policies have a bearing on the outcomes in the model. Thus, to match the time series observations in Doepke's model, the key is that relative costs of rearing skilled and unskilled children are changing over time due to the dynamics of the policies in different countries.

The paper by Becker, Murphy and Tamura (1990) contains a model in which there is an external effect from the accumulation of human capital. They do not have an explicit model of the transition. They show that, in their model, there are multiple steady states with different fertility and GNP growth rates. One of the steady states has high fertility and low GNP per capita growth (a Malthusian steady state) and the other has low fertility and a high GNP per capita growth (a modern steady state). They conclude that luck must play an important role in the transition, since it requires a move from one steady state to another.

In addition to that literature, another group of papers concentrates on models of fertility and the demographic transition, but those papers are not based on microfoundations through external effects between members of a family. Rather, they use reduced-form representations of preferences (e.g., utility for either expected future family income or the number of children) that do not correspond to either the Barro and Becker formulation or the

one analyzed here. Some of these papers do construct models with the possibility of a demographic transition (see Galor and Weil (1998, 1999)), while others study the effects of infant mortality on fertility (e.g., Kalemli-Ozcan, Ryder, and Weil (2000)). None of these papers contains quantitative implementations of the models, however, and they often obtain results that are at odds with models based on microfoundations, making their results difficult to interpret. Finally, other authors have estimated reduced-form representations of dynamic models of fertility choice using data from various historical episodes of the fertility transition (see e.g., Haynes, Phillips, and Votey (1985) and Eckstein, Mira, and Wolpin (1999)).

The older paper we are aware of, which adopts the “children as investment” approach to endogenous fertility is Neher (1971). This is a three period OLG model in which middle-age agents care for their current consumption, their consumption when old, and their children’s consumption two periods later, when the children are in their last period of life. The economy faces a neoclassical technology in which labor (of middle-age people) is the only variable factor. A share-alike ethic is assumed, according to which the output of each period is shared equally among the three generations alive. Neher concentrates on deriving a Golden Rule for fertility and on the fact that this rate of fertility is highly sensitive to how much middle-age agents discount their own future utility and that of their children. He argues that in general, one should expect overpopulation as parents underestimate the full social cost of bearing new children to the extent that this is realized two periods from now, when the generating parents are dead. Neher does point out something which is also apparent in our analysis: the establishment of more efficient private financial markets and the introduction of pension schemes that do not depend upon one’s own fertility should tend to reduce the demand for children. Since Neher’s (1971) paper, this theme has been taken up by a number of other authors, e.g., Ehrlich and Lui (1991), Azariadis and Drazen (1993), Zhang and Nishimura (1993), Raut and Srinivansan (1994), Chakrabarti (1999), among other. Interestingly enough, the very simple model we study here in which the external effect runs from parents to children, intergenerational transfers are endogenous, and capital accumulation is possible has not yet been considered, to the best of our knowledge. More often than not researchers have

assumed that the portion of labor income being transferred as a “pension” to the old parents is an exogenous parameter of the model. Nishimura and Zhang (1993) are an exception, as they endogenize this donation in a form which is similar to our noncooperative solution. Their analysis concentrates on the existence of endogenous oscillations in the fertility and saving rates and they are not concerned with matching stylized historical facts or with evaluating the impact that changes in mortality have on fertility rates.

The Ehrlich and Lui (1991) paper is particularly close to ours, at least in the direction of the external effect within the family. They look at the family as a mutual insurance mechanism in which both intergenerational transfers and altruism are at play. They concentrate not on the role of land in traditional societies but, instead, on human capital accumulation and its relationship with persistent growth. Parents invest in children’s human capital and, by assumption, are entitled to an exogenous fraction of their children’s earnings. Thus, they receive a transfer when old, which is, by assumption, strictly proportional to the wage bill of their, then working, offspring. To the extent that human capital can be accumulated without bounds, this generates a model of endogenous growth driven by interfamilial arrangements. Their model does not induce an equilibrium demographic transition, however. Hence, they enrich their model by introducing a specific version of the “children-as-consumption” hypothesis. In particular, they assume a “companionship function,” according to which parents receive utility from the number and quality of their children. The quantitative properties of these models are not studied in the paper.

Our work is therefore complementary with that of the aforementioned papers. We endogenize the degree of support that parents get from their children and consider both cooperative and noncooperative mechanisms for determining its level. This allows us to study the effects of a lack of commitment (by children for the level of support that they will offer parents) and compare the effects of alternative transfer arrangements. Thus, we can outline a formal model which may be able to capture the transition from tight to loose family arrangements that many anthropologists and sociologists say occurred during the same period in which the demographic transition took place.

### 3 The Basic Model

The basic version of the model concentrates on the impact of infant and child mortality on fertility choices. The setup is meant to capture the crucial features of a traditional agricultural society, with competitive markets for land and labor but without any form of technological progress. Adding exogenous growth in labor productivity does not alter any of the substantive conclusions about fertility in the model, and hence, it is not included here.

Agents live for a maximum of three periods, young ( $y$ ), middle-age ( $m$ ), and old ( $o$ ). At birth, individuals have an endowment of productive time equal to  $(0, 1, 0)$ .

#### 3.1 Fertility

People born in period  $t$  are capable of reproducing during period  $t + 1$ . They choose the number of per capita children, which we denote by  $f_{t+1}$ . If  $N_{t+1}^m$  middle-age people are alive,  $f_{t+1}N_{t+1}^m = N_{t+1}^y$  children are born in period  $t + 1$ .

Denote by  $\theta_t$  the amount of resources (current consumption) needed to rear one child. Given the level of fertility, total resources needed for rearing children are  $N_t^m \cdot \theta_t \cdot f_t$ .

A general specification for  $\theta_t$ , capturing both the consumption and the time cost of rearing a child, is

$$\theta_t = a + bw_t$$

where  $w_t$  is the wage rate in period  $t$ , and  $a$  and  $b$  are positive constants. In our analysis, we consider separately the two extreme cases in which  $a = 0$  and  $b = 0$ . While the qualitative results are the same, the quantitative response of the model to variations in mortality rate are affected.

#### 3.2 Mortality

A fraction of individuals alive in period  $t$  die at the end of that period. For old people, this fraction is always equal to one. For middle-age people, the mortality rate is assumed to be zero in this version of the model: hence, we

abstract from changes in life expectancy of these individuals. For the young, the mortality rate is given by  $m_t^y \in [0, 1)$ . Because of these assumptions, it follows that old and middle-age people in period  $t + 1$  are equal to one and  $f_t(1 - m_t^y)$  times the number of middle-age people in the previous period, respectively. Notice that  $m_t^y$  should be interpreted as the total mortality rate between birth and working age.

In the illustrative simulations reported in Section 5, we assume the length of a period to be approximately 20 years. This is obviously a gross simplification, which is made particularly inaccurate by our assumption of a constant survival rate of one between the second and third periods of life. European demographic data show that, since the end of the XVI until the middle of the XIX century, most gains in life expectancy are concentrated among adults as opposed to children. This change in life expectancy, population age structure, and incentive to have children cannot be captured by this simple version of our model and will have to be the object of further research.

Denote by

$$\pi_t = 1 - m_t^y$$

the exogenous process for the probability of survival of the young. Then  $\pi_t$  is the probability that a child born in period  $t$  reaches middle-age in period  $t + 1$ .

For future reference, total population at time  $t + 1$  is

$$N_{t+1} = N_{t+1}^y + N_{t+1}^m + N_{t+1}^o$$

and hence, the growth rate of population is

$$1 + g_t^n = \frac{N_{t+1}}{N_t} = f_{t-1}\pi_{t-1} \cdot \frac{1 + (1 + f_{t+1})f_t\pi_t}{1 + (1 + f_t)f_{t-1}\pi_{t-1}}.$$

Thus, along a balanced growth path associated with any constant survival probability, we have

$$1 + g^n = \pi \cdot f.$$

### 3.3 Production

In period  $t$ , total consumable output is

$$Y_t = F(L_t, K_t)$$

where  $K_t = K_{t-1} + N_{t-1}^m s_{t-1}$  is the total stock of land/capital (we assume zero depreciation) and  $L_t = N_t^m$  is the total labor supply from middle-age people. We make the simplifying assumption that the resources saved and invested to acquire new land or in accumulating new capital ( $s_t$ ) translate into new land/capital at a constant one-to-one rate. Notice that this implies both an innocuous choice of units in which land/capital is measured and the less innocuous assumption that returns to investment are constant and land/capital is homogenous and can be accumulated indefinitely. Introducing decreasing returns in the investment technology would probably make the model more realistic but add little insight to our analysis. The main difference would be that, with decreasing returns, the size of the economy could not grow forever and a steady state should be reached in the absence of some form of technological progress. With our assumption of constant returns, instead, a balanced growth path is reached. Assume that  $F(K, L)$  is concave, homogenous of degree one, and increasing in both arguments. As usual, set  $f(k) = F(K/L, 1)$ .

The aggregate resource constraint is

$$Y_t \geq N_t^o \cdot c_t^o + N_t^m \cdot [c_t^m + s_t] + N_t^y \cdot \theta_t.$$

Other definitions and accounting identities, used throughout the paper, are

$$k_t = \frac{K_t}{L_t} = \frac{K_t}{N_t^m}$$

$$x_t = \frac{K_t}{N_t^o} = \frac{K_{t-1} + N_{t-1}^m s_{t-1}}{N_{t-1}^m} = k_{t-1} + s_{t-1} = \pi_{t-1} f_{t-1} k_t.$$

Along a balanced growth path, the stock of capital per worker, individual saving, and fertility must satisfy

$$k^* = \frac{s^*}{\pi f^* - 1}.$$

### 3.4 Utility Function and Budget Constraints

People receive utility from their own consumption and from that of their parents. The young need parental attention to consume and survive. For

simplicity, we assume that consumption when young does not affect life time utility. We assume that there is no utility from leisure; that labor supply of both the young and old is zero, and we normalize by setting the labor supply of the middle-age person at one. Given these assumptions, the utility function of an individual born in period  $t - 1$  (middle-age in period  $t$ ) is given by  $U_{t-1} = u(c_t^m) + \eta u(c_t^o) + \delta u(c_{t+1}^o)$  where  $u(\cdot)$  has all the standard properties of a concave utility function. The parameters  $\delta$  and  $\eta$  are in  $(0, 1)$ :  $\delta$  is the discount factor, while  $\eta$  reflects the value that children place on the consumption of their parents.

There is no budget constraint in the first period of life; the young are taken care of by their parents. Denote by  $w_t$  the wage rate per unit of labor time, and let  $d_t^i$  denote the per capita donation from middle-aged individuals to their parents. We have

$$d_t^i + c_t^m + s_t + \theta_t f_t \leq w_t$$

when people are middle-age, and

$$c_t^o \leq \sum_{i=1}^{N_t^m} d_t^i + R_t x_t$$

when people are old. Recall that  $n_t^m = N_t^m/N_t^o = f_{t-1}\pi_{t-1}$  is the number of surviving children per old person and  $R_t$  is the rate of return on capital. Notice that the rate of return  $R_t$  measures the consumable output due to land/capital and is net of the (undepreciated) stock of land/capital  $K_t$ . Because of zero depreciation, we also assume that old people inherit the existing stock of land/capital from their dead parents.

### 3.5 Games of Giving

In period  $t$ , each one of the  $i = 1, 2, \dots, N_t^m$  surviving middle-age agents donates some consumption to his or her parents. We consider two solutions to the gift-giving game. In the **cooperative** solution middle-age agents maximize the simple sum of their (equally weighted) utilities by choosing the total consumption of their parents and sharing the burden equally. In the

noncooperative solution each middle-age agent takes the gift of his siblings as given, and the equilibrium concept is Nash with symmetry.

We associate the cooperative solution to environments in which the organization of economic activity and social norms are such that both parental authority over own children and reciprocal control among siblings are strong enough to enforce cooperative behavior. What we have in mind are traditional agrarian communities where patriarchal families constitute the backbone of society. In such environments, family land and other possessions (animals, tools, seeds) are transferred from parents to children in a relatively egalitarian form. Further, children, by living nearby and cultivating the land left them by the parents, are able to monitor each other's behavior and enforce cooperative rules. When such traditional structures break down, people move apart and engage in economic activities other than working the family land. Both parental authority over children and the extent to which the latter can monitor each other are greatly reduced. Noncooperative behavior appears as a much more reasonable assumption in such circumstances.

In the cooperative equilibrium, middle-age agent  $i$  chooses donation  $d_t^i$  to solve

$$\max_{d_t^i} u^i(w_t - \theta_t f_t - d_t^i - s_t) + \eta u^i(n_t^m d_t^i + R_t K_t / N_t^o).$$

First-order conditions yield

$$u'^i(c_t^m) = n_t^m \eta u'^i(c_t^o).$$

Note that this is the same first-order condition that would be obtained through joint maximization of the sum of utilities of the siblings.

In the noncooperative equilibrium, middle-age agent  $i$  chooses donation  $d_t^i$  to solve

$$\max_{d_t^i} u^i(w_t - \theta_t f_t - d_t^i - s_t) + \eta u^i \left( \sum_{j \neq i, j=1}^n d_t^j + d_t^i + R_t K_t / N_t^o \right).$$

First-order conditions yield

$$u'(c_t^m) = \eta u'(c_t^o).$$



Whenever  $\pi_{t-1}f_{t-1} = n_t^m > 1$ , the cooperative solution entails a higher individual donation and, ceteris paribus, higher consumption for the old people.

### 3.6 Life Cycle Problem and First-order Conditions

The planning problem for a middle-age individual is

$$\begin{aligned} \max_{s_t, f_t, d_t^i} & u^i c_t^m + \delta u^i c_{t+1}^o + \eta u^i c_t^o \\ \text{s. t.} & c_t^m + \theta_t f_t + d_t^i + s_t \leq w_t \\ & c_{t+1}^o \leq \sum_{i=1}^{n_{t+1}^m} d_{t+1}^i + R_{t+1} x_{t+1}. \end{aligned}$$

First-order conditions with respect to  $s_t$ ,  $f_t$ , and  $d_t^i$  yield

$$u^i c_t^m = \delta u^i c_{t+1}^o \frac{\partial c_{t+1}^o}{\partial s_t}, \quad (1)$$

$$u^i c_t^m \theta_t = \delta u^i c_{t+1}^o \frac{\partial c_{t+1}^o}{\partial f_t}, \quad (2)$$

$$u^i c_t^m = \eta u^i c_t^o n_t^m, \quad (3)$$

$$u^i c_t^m = \eta u^i c_t^o. \quad (4)$$

Conditions (1) and (2) are traditional intertemporal conditions for investment decisions. They equate the ratio of discounted marginal utilities to the rate of return on the particular investment project at hand. Notice, though, that a strategic component is introduced here. This is because donations from children to parents take place within individual families. To the extent they are aware of the strategy (cooperative or not) children will follow in determining  $d_{t+1}$ , it is rational for parents to take into account the impact that a variation in the amount of savings and in the number of siblings may have on the total donation they will receive from their children when old. Hence, the terms  $\frac{\partial c_{t+1}^o}{\partial s_t}$  and  $\frac{\partial c_{t+1}^o}{\partial f_t}$  appear on the right-hand sides of (1) and (2) instead of the “competitive” rates of return  $R_{t+1}$  and  $\pi_t d_{t+1}$ . Finally, equations (3) and (4) are the first-order conditions for the choice of donation in the cooperative case and noncooperative case, respectively.

## 4 Log Utility and Cobb-Douglas Production

We now solve the model explicitly for the case of logarithmic utility and Cobb-Douglas production functions.

Set  $u(c) = \log(c)$  and  $F(K, L) = AK^\alpha L^{1-\alpha}$ . Then

$$\begin{aligned} \max_{f_t, s_t, d_t^i} \log c_t^m + \delta \log c_{t+1}^o + \eta \log c_t^o \\ \text{s. t. } c_t^m + \theta_t f_t + d_t^i + s_t \leq w_t \\ c_{t+1}^o \leq \prod_{i=1}^m d_{t+1}^i + R_{t+1} x_{t+1}. \end{aligned}$$

### 4.1 Cooperative Equilibrium

From the first-order condition (3) and the budget constraint, per capita donation and consumption when agents are old are, respectively,

$$d_t = \frac{\eta}{1+\eta} w_t - \theta_t f_t - s_t - \frac{R_t x_t}{n_t^m (1+\eta)}$$

and

$$c_t^o = \frac{\eta}{1+\eta} n_t^m w_t - \theta_t f_t - s_t + R_t x_t.$$

Hence, the relevant derivatives are

$$\begin{aligned} \frac{\partial c_t^o}{\partial s_{t-1}} &= \frac{R_t \eta}{1+\eta} \\ \frac{\partial c_t^o}{\partial f_{t-1}} &= \frac{\pi_{t-1} \eta}{1+\eta} w_t - \theta_t f_t - s_t. \end{aligned}$$

The equality of rate of returns condition yields

$$\theta_t R_{t+1} = \pi_t w_{t+1} - \theta_{t+1} f_{t+1} - s_{t+1}. \quad (5)$$

This economy has one endogenous state variable, the capital/labor ratio  $k_t = K_t/N_t^m$ , and one exogenous state variable, the survival probability  $\pi_t$ . Hence,

the equilibrium dynamics should be summarized by some function  $k_{t+1} = h_{\pi_t}(k_t)$ , where  $h_{\pi_t}$  is a one-parameter family of continuous functions from the positive reals into themselves. If, given initial conditions, there is only one such  $h_{\pi}$  for any given  $\pi$ , the competitive equilibrium is unique. Notice that, given the initial condition  $k_t$  which determines  $w_t$  and hence  $\theta_t$ , condition (5) involves only tomorrow's capital/labor ratio and exogenous parameters. This means that, given the initial conditions  $k_t$  and  $\pi_t$ , tomorrow's capital/labor ratio is uniquely determined and equilibria are unique if (5) has a unique solution. In such circumstances and for a constant  $\pi$ , the economy converges asymptotically<sup>6</sup> to a constant capital/labor ratio  $k(\pi)$  compatible with (5). Associated with each such constant  $k(\pi)$ , there is a balanced growth rate  $\pi \cdot f(\pi)$ , where  $f(\pi)$  is the fertility rate associated to  $\pi$ .

First-order conditions for the determination of saving and fertility in period  $t$  can be manipulated by replacing in either of them the optimal amount of donation and the equality of rates of return condition to get

$$s_t + \theta_t f_t = \frac{1}{1 + \delta + \eta} (\delta w_t + R_t k_t - (1 + \eta)k_t).$$

Hence, aggregate investment  $I_t$  (the sum of investment in capital and in children) is

$$s_t + \theta_t f_t = \frac{1}{1 + \delta + \eta} (\delta f(k_t) - (1 + \eta)k_t) = g(k_t). \quad (6)$$

Notice that, in principle,  $g(k_t)$  is a nonmonotone function of  $k_t$ , as it satisfies  $g(0) = g(\bar{k}) = 0$  for some finite value  $\bar{k} > 0$ . Per capita saving is a non-monotone function of per capita stock of capital. An economy with a large endowment of capital relative to labor will accumulate little or no additional capital and expand the population to bring down  $k$  toward the long-run efficient ratio. This seems to fit well with observed experience.

In practice, the intertemporal equilibrium is computed this way. Use  $K_t, L_t$  to determine  $y_t = f(k_t)$  and  $g(k_t) = I_t$ . Given  $I_t$  and  $k_t$ , the three equations

$$I_t = s_t + \theta_t f_t$$

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<sup>6</sup> Convergence takes place in one period when  $b = 0$ .

$$k_{t+1} = \frac{k_t + s_t}{\pi_t f_t}$$

and

$$f'(k_{t+1}) = \frac{\pi_t}{\theta_t} [f(k_{t+1}) - k_{t+1}f'(k_{t+1}) - g(k_{t+1})] \quad (7)$$

determine  $f_t$ ,  $s_t$ , and  $k_{t+1}$ . The last one, together with mortality rates and initial conditions, determines the stock of capital and the structure of the population tomorrow. Hence, for given initial conditions, this system is determinate and a unique equilibrium path exists as long as the condition (7) has only one solution.

We next check if the rate of return condition can be satisfied by more than one value of  $k^*$ . Algebraic manipulation of (7) gives

$$f'(k^*) \left[ 1 + \frac{\pi}{\theta} k^* \right] = \frac{\pi(1+\eta)}{\theta(1+\delta+\eta)} [f(k^*) + k^*].$$

The left hand side is a nonmonotone function: decreasing near zero until a minimum, then increasing again without bound. The right-hand side is monotone increasing. So, in principle, for each set of exogenous parameters, more than one desired capital/labor ratio could exist. This means that, potentially, the system has multiple equilibria as, for a given exogenous sequence of  $\pi_t$ , it may randomly jump back and forth between these two balanced growth paths. But functional forms for which more than one solution to (7) exist are not easy to find. In the Cobb-Douglas case, we have

$$A\alpha k^{\alpha-1} = \frac{\pi}{\theta} \left[ A(1-\alpha) - \frac{\delta}{1+\delta+\eta} \right] k^\alpha + \frac{1+\eta}{1+\delta+\eta} k.$$

Here the LHS is monotone decreasing. The RHS is monotone increasing for reasonable parameter values, i.e., for  $(1-\alpha) > \frac{\delta}{1+\delta+\eta}$ . Hence, in the Cobb-Douglas case, we expect only one, asymptotically stable balanced growth path and a unique equilibrium for every initial condition and sequence of exogenous survival probabilities  $\pi_t$ .<sup>7</sup>

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<sup>7</sup> The argument given here and in the previous paragraph treats the cost of raising children  $\theta$  as an exogenous parameter instead of an endogenous value (via the wage rate). It is straightforward to verify, though, that all results remain true when the endogenous nature of  $\theta$  is taken into explicit account.

## Population Growth in a Malthusian Economy

We call our economy Malthusian because of the hypotheses upon which it is built. There is no technological progress, and aggregate production displays constant returns to scale on land/capital and labor. Production of children also displays constant returns, and children are seen as an investment good by their parents insofar as they provide future labor to work the land. These were Malthus' original assumptions. (See Malthus (1798).) To understand the Malthusian predictions of this simple model, begin by noticing that along a balanced growth equilibrium, income per capita remains constant as the population reproduces at the same rate as the stock of capital, and there is no technological progress. On the other hand, an increase in  $\pi$  (reduction in mortality rates) always decreases the capital/labor ratio because the desired ratio is determined by condition (7) and any increase in  $\pi$  increases the rate of return on fertility. Higher fertility means a larger middle-age group next period and a lower capital/labor ratio. To each  $k^*(\pi)$ , the model associates a balanced growth rate  $g^*(\pi) = \pi f^*(\pi)$  which is increasing in  $\pi$ , as we can verify by taking derivatives. Notice that while  $g^*(\pi)$  is increasing in  $\pi$ ,  $f^*(\pi)$  is not. In fact, with a certain amount of algebra, we can prove that fertility is always decreasing in  $\pi$  in our model.<sup>8</sup> This is a distinguishing feature of our model, which is consistent with probably the most important and universal feature of historical demographic transitions. Nevertheless, while fertility decreases with infant mortality, it does so less than proportionally. Hence, as mortality decreases, population expands at higher and higher rates. This second prediction is only partially consistent with the historical experience: as mortality declines the growth rate of population increases for a while, sometime for a century or more, but eventually decreases. The simple version of our model fails to generate an eventual decrease in the growth rate of population. This is a relevant limitation, which we discuss extensively later. The capital stock expands likewise, but at a lower rate. Consequently, per capita labor productivity decreases (as technological change is absent) when population increases.

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<sup>8</sup> We are grateful to an anonymous referee for showing this to us. Details of algebra available upon request.

In summary, intertemporal equilibria constructed by moving from one balanced growth path to another with  $\pi_t$  increasing have the following Malthusian properties.

1. Fertility decreases, but the growth rate of the population increases.
2. The capital/labor ratio decreases, together with per capita income and consumption.
3. The wage rate decreases, and the rate of return on capital increases.

These properties are illustrated numerically in the calibration exercise we report in Section 5.

## 4.2 Noncooperative Equilibrium

Next, we examine the properties of the model when siblings play the noncooperative game as outlined above. We have already discussed the first-order conditions affecting the level of donations from children to parents. Using those first-order conditions and the budget constraint we can compute middle-age donation and old-age consumption:

$$d_t^i = \frac{1}{n_t^m + \eta} \eta^i w_t - \theta_t f_t - s_t - R_t x_t$$

and

$$c_t^o = \frac{\eta}{n_t^m + \eta} n_t^m w_t - \theta_t f_t - s_t + R_t x_t .$$

Hence, the relevant derivatives are

$$\frac{\partial c_t^o}{\partial s_{t-1}} = \frac{R_t \eta}{n_t^m + \eta}$$

and

$$\frac{\partial c_t^o}{\partial f_{t-1}} = \frac{\pi_{t-1} \eta}{(n_t^m + \eta)^2} \eta^i w_t - \theta_t f_t - s_t - R_t x_t .$$

Notice that, contrary to the cooperative case, the impact of increased fertility upon consumption in old-age may now be negative. This occurs when the

desired donations are, in fact, also negative. For this to be the case the return on accumulated capital must have become a large enough share of national income that the equalization of marginal utility of consumption between old parents and their offsprings requires a transfer in the opposite direction. While we do not explore in this paper the implications of this feature of our model, it should be kept in mind when thinking at the impact that an increase in the efficiency and availability of capital markets would have on fertility and intergenerational transfers within the family.

Equality of rates of return yields first

$$R_{t+1} = \frac{\pi_t}{\theta_t(\eta + n_{t+1}^m)} \eta(w_{t+1} - \theta_{t+1}f_{t+1} - s_{t+1}) - R_{t+1}x_{t+1}$$

and then, after simplification,

$$R_{t+1} = \frac{\eta\pi_t}{\theta_t(\eta + n_{t+1}^m) + \pi_t x_{t+1}} (w_{t+1} - \theta_{t+1}f_{t+1} - s_{t+1}) \quad (8)$$

Next we use (1) and (8) to obtain an explicit solution for aggregate investment  $I_t = \theta_t f_t + s_t$ :

$$I_t = \frac{\delta\eta(w_t n_t^m + R_t x_t) - (\eta + n_t^m)(\eta + n_{t+1}^m)k_t}{(\eta + n_t^m)(\eta + n_{t+1}^m) + \delta\eta n_t^m}.$$

Notice that  $w_t n_t^m + R_t x_t = n_t^m f(k_t)$ , where  $f(k_t)$  is output per middle-age worker. So this is output per member of the old generation. Define the weight  $\gamma_t \in (0, 1)$  as

$$\gamma_t = \frac{\delta\eta n_t^m}{(\eta + n_t^m)(\eta + n_{t+1}^m) + \delta\eta n_t^m}.$$

Per capita total investment of the middle-age portion of the population can be written as

$$I_t = \gamma_t [f(k_t) + k_t] - k_t \quad (9)$$

Inspection of (9) and (8) shows that uniqueness of the interior balanced growth path is no longer a straightforward matter here, as it was in the cooperative model. In fact, the function  $f_{AI}(k^*)$  that solves (9) at a steady

state is nonmonotone and the same is true for the solution to (8). By manipulating functional forms one can construct examples in which, beside the degenerate steady state at which  $k^* = f^* = 0$ , there are one, two or even three interior balanced growth paths.<sup>9</sup> Hence, the noncooperative version of our model is able to display dependence upon initial conditions and multiple balanced growth paths, some asymptotically stable other unstable.

We will not dwell further on these properties of our model here, for two reasons. On the one hand, we do not believe such multiplicities are empirically relevant for understanding demographic transitions. On the other hand, in the log utility and Cobb-Douglas production case, for values of  $\alpha \leq .5$  uniqueness holds. As  $\alpha = .5$  is the highest value we are interested in, multiplicity is not an issue for us. In summary, for the set of parameter values we are consider the noncooperative model behaves very much like the cooperative one: there exists a unique interior balanced growth path to which, when  $\pi_t = \pi$  for all  $t$ , all equilibria converge. A word on why we are not interested in studying multiple equilibria in this paper is in order. While we find multiple equilibria an interesting theoretical twist<sup>10</sup> we cannot avoid seeing them as a weak explanation for the existence of fertility differentials that are persistent over time, and for the demographic transition in particular. The demographic transition, which is our main concern here, is a dynamic phenomenon, common to almost all countries and with remarkably similar patterns of evolution. We find it highly implausible to interpret such historical movements as the outcome of a worldwide coordination in jumping, in different historical periods but just at the right moment in each country, from one equilibrium level of fertility to another, following some not well identified sunspot signal.

While the model cannot be solved analytically, a numerical investigation of its properties can be carried out. Simulations reported in the following

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<sup>9</sup> Again, thanks to an anonymous referee for pointing out that our initial assertion based on numerical simulations, according to which a unique interior balanced growth path existed, was not true in general and for providing us with a straightforward proof of the result.

<sup>10</sup> The presence of which, in this model, shows that one does not need to adopt complicated production externalities if all one wants are multiple equilibria in a competitive model of endogenous fertility.



sections concentrate, in particular, on the dependence of the fertility rate, the population growth rate, and the capital/labor ratio on the survival probability  $\pi_t$ . Our main findings are summarized next.

1. The qualitative predictions of the model are the same as in the cooperative case. In particular, as  $\pi$  increases, fertility decreases,  $k$  decreases, and the growth rate of the population increases.
2. The quantitative properties are remarkably different. At the same parameter values the equilibrium level of fertility is substantially lower, and that of the capital labor ratio higher than in the cooperative case.
3. Similarly, fertility drops much faster as the survival probability increases yielding a growth rate of the population that is barely increasing over the historically relevant range of values for  $\pi$ .
4. In particular, the parameter values at which the noncooperative model replicates the fertility behavior of the period 1561-1661 are substantially different from those for the cooperative one.

## 5 Calibrating and Simulating the Model

In this section, we present a calibration of the model outlined above and use it to run a few simple numerical experiments. One word of caution seems appropriate on the interpretation of the results to be presented. The simple model discussed in this paper is meant to focus on two points, which have been left at the margin of the economic literature on the demographic transition: the impact of exogenous changes in the IMR upon, and the relevance of intergenerational sharing agreements within the family for, the determination of aggregate fertility. To properly concentrate attention on these issues, we abstract from the overall evolution of life expectancy<sup>11</sup>, from the impact of technological progress (either endogenous or exogenous), from intergenerational transfer agreements implemented outside the family (e.g., pensions,

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<sup>11</sup> In fact, with a three-period model in which each period equals 20 years, we are forgoing any ability to capture the demographic evolution of the XX century.

education, public health), and finally, from the development of capital markets. Our numerical experiments are therefore to be read as possible answers to three simple questions. From a historical standpoint, do the comparative static responses of fertility and population growth to increases in young-age survival rates appear reasonable? In the absence of any increase in per capita income, can a persistent drop in the infant mortality rate, similar to the one experienced in Europe in the second half of the XIX century, bring about a historically reasonable drop in fertility and increase in population growth rate? Does a shift from cooperative to noncooperative behavior entail a significant quantitative difference in the model's response to these perturbations? We interpret the numerical results as implying a positive answer to each question.

We calibrate the model in such a way that, when the survival probability  $\pi$  is constant and equal to its average for England from 1561 to 1661, the model yields a growth rate of population equal to the observed average in the same country over the same period. This implies a fertility rate that is slightly lower than the one historically observed between 1561 and 1661, as we assume a mortality rate of zero between the second and third periods of life, which, together with the mortality rate of 0.100 percent at the end of the third period, yields a Crude Death Rate (CDR) slightly lower than the average historical value. Next, we use this calibrated version of the model to carry out some numerical experiments.

First, we carry out a comparative static exercise by computing the values of fertility, population growth, capital/labor ratio, and average labor productivity as  $\pi$  increases from its historical lower bound (about 0.4) to its maximum of 1.0. Second, we compute the model's predictions for the dynamics of response to a one-time, unexpected increase in infant mortality. This might correspond to the response of a Malthusian economy to the onset of a plague or famine similar to those for which historical records are available. These computations are done for both the cooperative and non-cooperative versions of the model. For comparative purposes, and to justify the statements made in Sections 1 and 2, we also include a version of the first simulation for a similarly calibrated version of the Barro and Becker model.

## Calibrating the Malthusian Economy

To avoid repetitions, we use the cooperative model to describe the procedure used for calibration. We have adopted the same logic for the noncooperative model, with the obvious changes in the form of the equilibrium conditions. For the cooperative model we have

$$s_t + \theta_t f_t = \frac{1}{1 + \delta + \eta} A \delta k_t^\alpha - (1 + \eta) k_t \quad (10)$$

$$A \alpha k_{t+1}^{\alpha-1} = \frac{\pi_t}{\theta_t} A \left[ 1 - \alpha - \frac{\delta}{1 + \delta + \eta} \right] k_{t+1}^\alpha + \frac{1 + \eta}{1 + \delta + \eta} k_{t+1} \quad (11)$$

$$k_{t+1} = \frac{k_t + s_t}{\pi_t f_t} \quad (12)$$

Given  $k_t$ ,  $\pi_t$  and a set of parameters  $\alpha, \eta, \delta, A$ , and  $\theta_t = a + b w_t$ , equations (10) – (12) determine the three endogenous variables  $f_t, k_{t+1}, s_t$ . To determine the appropriate values for the six parameters, we need to make some assumptions linking our model to observable data. A natural benchmark is the case in which the cost of rearing children consists mainly of parents' time, i.e.  $a = 0$ . All the simulations reported, but one mentioned below, have been computed under this assumption. Results are not changed, in fact sometime improved, by the choice of  $a > 0$  and  $b = 0$ .

Fairly detailed population, fertility, and mortality data are available for England since 1541, at intervals of either five or ten years, from Mitchell (1978) and Wrigley and Schofield (1981). Data on agricultural productivity, wages, and factor shares are available from Clark (1999) and Hoffman (1996). We use these data to calibrate our model. We proceed as follows.

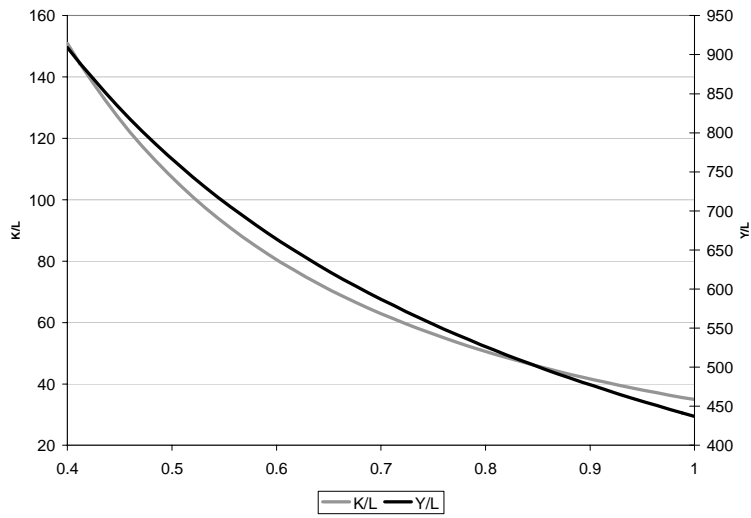
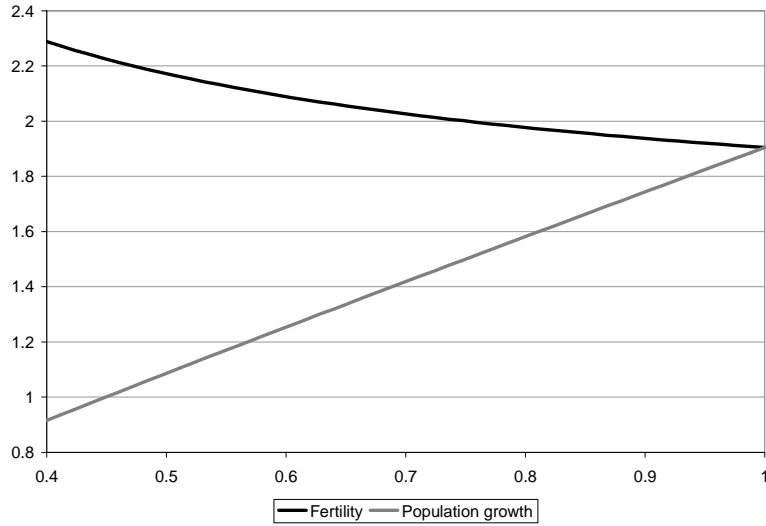
1. We assume that each period of our model corresponds to 20 calendar years.
2. From the data, we compute the probability of survival  $\pi_t$  at age 20 for each intermediate period  $t = 1541 - 1561, 1561 - 1581, \dots, 1961 - 1981$ .
3. From data in Clark (1999) and Hoffman (1996) we compute  $\alpha = 0.5$  and  $A = 74.00$ .

4. We set  $\delta = (1.05)^{-20} = 0.3768$ .
5. From data, we compute the average survival probability ( $\pi = 0.523$ ) and the average population growth factor ( $\pi f = 1.1$ ) during the five periods comprised within the calendar years 1541-1641.
6. We calibrate  $b$  and  $\eta$  in such a way that our model generates a steady state growth rate of population equal to 1.1 when the survival probability is kept constant at  $\pi = 0.523$ . This procedure gives  $b = 0.105$ , and  $\eta = 0.185$ .
7. We set initial conditions according to three criteria. Total population in the first period is equal to its historical value in 1541 (about 4.2 million). Its composition is such that  $N_{1541}^y = 1.1/(1 - 0.523)N_{1541}^m$  and  $N_{1541}^m = 1.1N_{1541}^o$ . We choose the aggregate capital stock  $K_{1541}$  in such a way that  $k_{1541} = 100$ .

The first experiment we run with the model is to compute the changes in steady state (balanced growth) values of fertility, population growth rate, capital/labor ratio, and labor productivity as a function of the survival probability,  $\pi$ , in the interval  $[0.4, 1.0]$ . These changes are reported in Figures 4a and 4b.<sup>12</sup> The interpretation of these figures is straightforward. Fertility decreases rapidly as  $\pi$  increases, from an average of 4.6 children per woman when  $\pi$  is at its historical level in 1500 to about 3.8 children per woman when  $\pi$  reaches a value of one. In spite of this decrease in fertility, the population growth rate increases continuously, reaching levels of around 80 percent per period (20 years) at high levels of  $\pi$ . Capital per worker and hence income per capita decrease throughout this range even as the total stock of capital/land increases. As mentioned earlier, these are the basic Malthusian properties of our model. Notice that the population growth rate keeps increasing with  $\pi$  even if per capita fertility decreases and reaches high values when infant mortality is near zero. This second prediction is highly counterfactual. The population growth rate first increased and then eventually decreased as infant and youth mortality decreased during the second half of the XX century.

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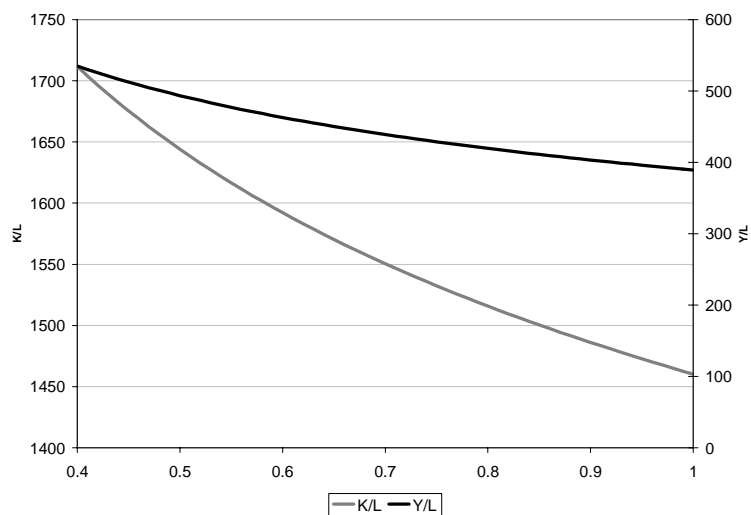
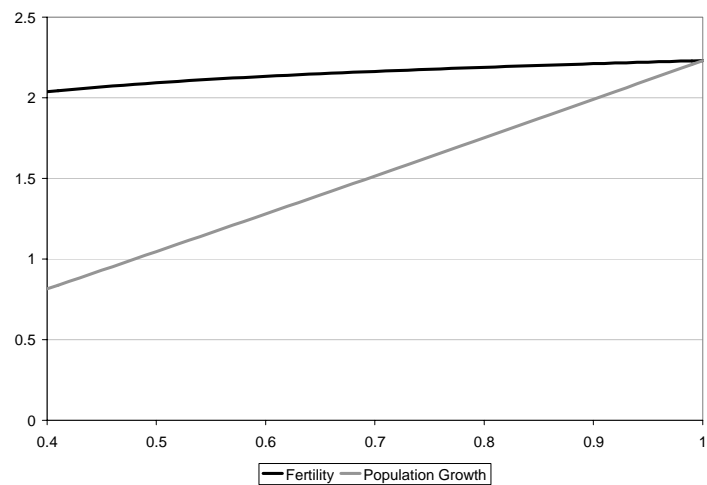
<sup>12</sup> Similar, in fact, quantitatively better, results obtain when  $a > 0$  and  $b = 0$ .



Figures 4a, 4b: The Cooperative Model

The comparative statics properties of the Barro and Becker model are less satisfactory. First, the population growth rate at high levels of  $\pi$  is extremely high in all the examples we calculated. More important: fertility

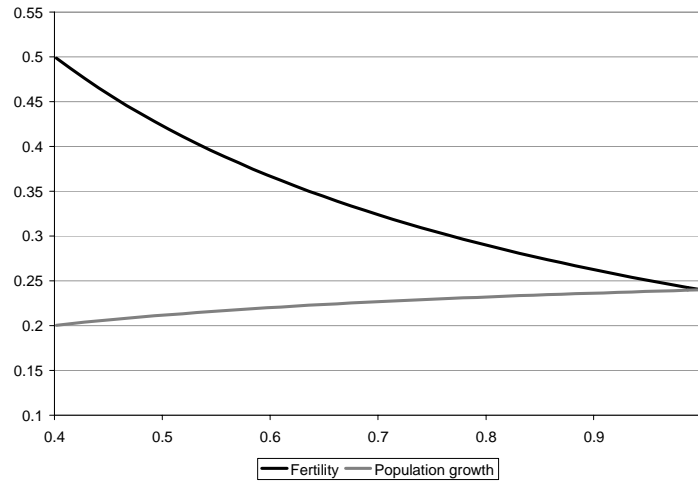
is also increasing (and strongly so) as a function of infant mortality.<sup>13</sup> Figures 4c and 4d show a typical set of calculations for a calibrated version of the Barro and Becker model when  $a = 0$ . For this choice of parameters, fertility increases rapidly as a function of the infant survival rate.

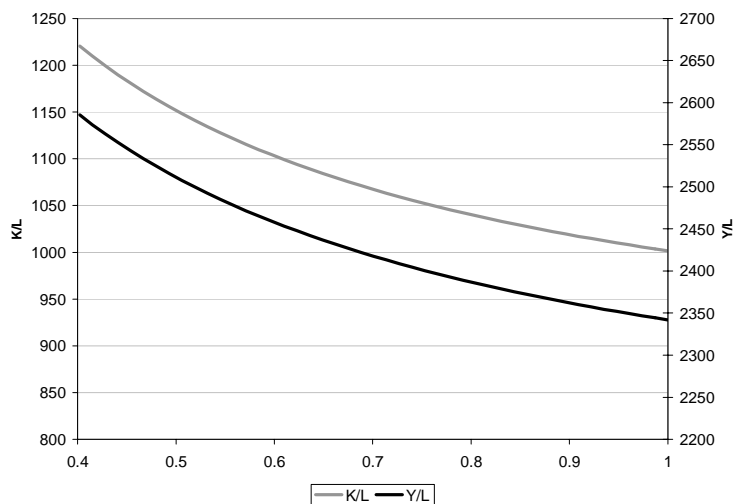


<sup>13</sup> When the cost of rearing children is fixed, that is,  $b = 0$ , fertility is increasing at some parameter values, decreasing at others, and even nonmonotone at some other times. Population growth rates are always high and increasing with  $\pi$ .

Figures 4c, 4d: The Barro and Becker Model

Figures 5a and 5b report the results from the same exercise for the noncooperative model. The same parameters are used here as in Figures 4a and 4b. While the qualitative behavior of the model follows that of the cooperative version, the quantitative results are strikingly different. Fertility is barely at 1 child per woman when the survival rate is  $\pi = 0.4$  and decreases to 0.5 as the survival rate goes to 1. As a consequence, the population growth rate is practically constant (and negative) as  $\pi \cdot f$  ranges from 0.4 to 1. Similarly, the capital/labor ratio is not only larger than in the cooperative case (1,200 vs. 100), it also decreases a lot less. Moreover, in the cooperative case, the balanced growth ratio  $K/L$  drops by more than 50 percent as  $\pi$  increases from 0.4 to 1 while it drops of about 10 percent when people behave noncooperatively.

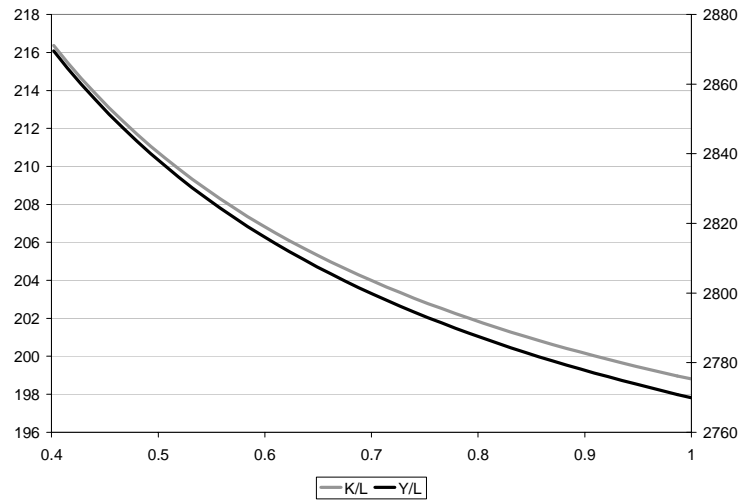
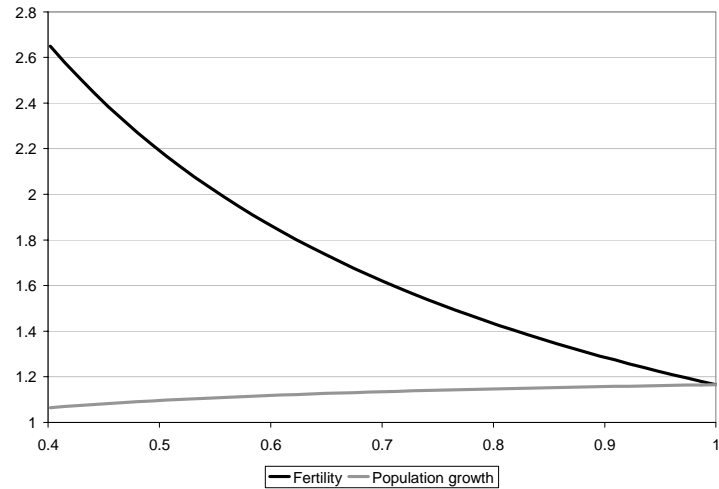




Figures 5a, 5b: The Non-Cooperative Model; “cooperative calibration”

For the sake of comparison, we report the results for a different calibration of the noncooperative model in Figures 5c and 5d. In this case, we have calibrated the model in such a way that the noncooperative model replicates the historically observed fertility and population growth rates when  $\pi = 0.52$ . The new set of parameter values is  $\alpha = 0.42$ ,  $\delta = 0.36$ ,  $A = 304.00$ ,  $\eta = 0.92$  and  $b = 0.005$ . In this calibration the drop in fertility is much more substantial than in the cooperative case, and the increase in the population growth rate is almost negligible. In fact, when the survival probability reaches contemporary values, fertility per woman is about 2.3 children and the population growth rate is 16 per cent every 20 years. For the sake of comparison, over the 20-year period from 1965 to 1985, the European population (as defined in the data of Figures 1 and 2) grew about 9 percent, while from 1970 to 1990 it grew about 7 percent. Thus, this version of the model mimics the beginning and end of the Demographic Transition quite well. What is missing in the model, however, is the nonmonotone behavior of the population growth rate (first increasing then decreasing) which we observe in the demographic transition.





Figures 5c, 5d: The Non-Cooperative Model; non-cooperative calibration

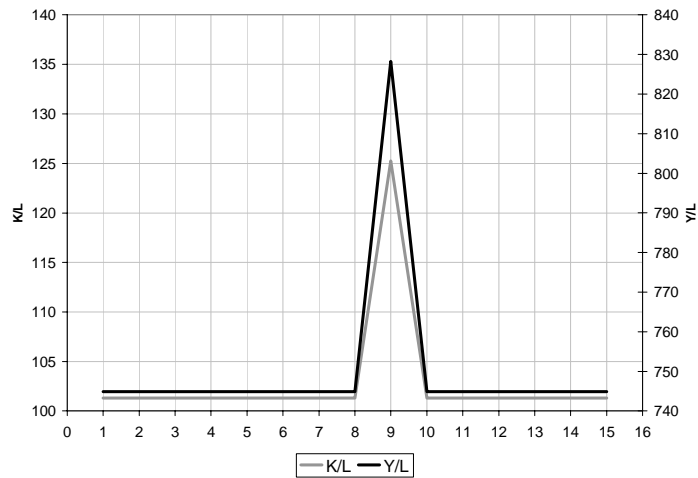
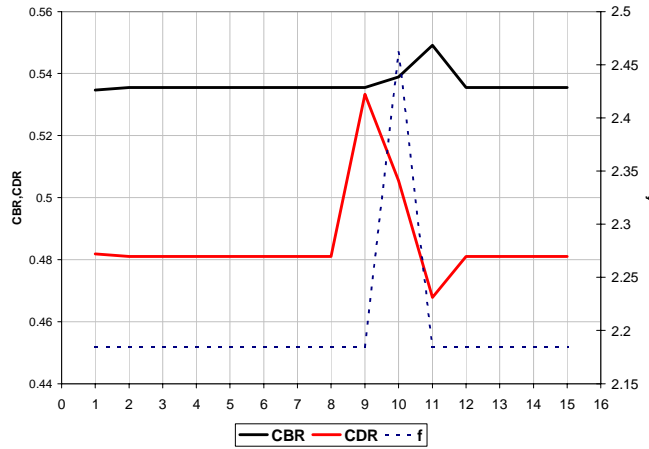
The second experiment is to calculate the predicted response to a one-time, short-term increase in the infant mortality rate. This mimics what would happen in a plague or a famine in our model economy if these shocks were unexpected and affected only the children's mortality rate. For this experiment, we held  $\pi$  at its steady state value of 0.523 and then decreased

it for one period (20 years). This was followed by a return to the steady state level. We assume the drop is unexpected so that, in the period of the shock, fertility cannot react to the sudden increase in mortality. From the data from the 16th to 18th centuries, a large famine or epidemic increased the infant mortality rate between 15 and 20 percent. For example, the infant mortality rate rose from 168 for the period from 1661 to 1671 to 202 for the period from 1671 to 1681 and then fell back to 174 from 1681 to 1691. Hence, we have carried out our experiment assuming that the unexpected shock increases children's mortality by 20 percent. Results are reported in Figures 6a and 6b.<sup>14</sup>

The overall response of fertility in the model is consistent with historical records: when mortality increases, fertility reacts with a lag and then increases more than proportionally, causing the crude birth rate to rise significantly. Parallel to this, the capital/labor ratio increases, leading to several consecutive decades of high labor productivity, high wages, and high income per capita. Subsequently, the increase in population that follows the shock brings the system back to the initial condition. Notice that the amount by which real wages first increase and then decrease in our model is small (about 10 to 12 percent) relative to historical values observed in Europe in the immediate aftermath of a plague. Historical variations were on the order of 25 to 40 percent with peaks of 50 percent in certain instances. (See, e.g., Phelps-Brown and Hopkins (1981) for detailed data.) Still, historical plagues and famines affected every age group and brought about reductions in the total population of between 20 and 40 percent. In our case, the reduction in total population is much smaller (about 6 or 7 percent) since only the young generation is assumed to be affected. Hence, our model seems to incorporate an elasticity of wages to aggregate employment which is stronger than that observed in historical data.

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<sup>14</sup> In this simulation we set  $b = 0$  and  $a = 27$ .



Figures 6a and 6b: Reaction to a one period mortality shock

As can be seen in Figure 6a and 6b, although mortality is increased dramatically, the crude birth rate is only slightly affected after the initial surprise because the offsetting effects on fertility and mortality are almost exactly equal. In the model, the increase in the CBR is quite modest, rising by about 5 percent over a 40-year period. In the data, a similar change is seen. The CBR per year in England rose from 28.91 per 1,000 in 1671 to 32.06 (about 10 percent) before falling back to 28.48 by 1706. Thus, this

simple model accounts for about 50 percent of the changes over this period in the data. No doubt, including the effect of deaths in other age groups would improve this accounting dramatically. The crude death rate increases from 26.25 to 32.14 per year in the data, or about 20 percent and only from 0.48 to 0.53, or about 11 percent, in the model.

## 6 Conclusions

We have studied a simple dynamic general equilibrium model of fertility and saving in an economy with overlapping generations and capital accumulation. To simplify, we have abstracted from technological progress and government intervention. From a theoretical perspective, the distinguishing feature of our model is the assumption that parents reproduce only to guarantee themselves some economic support in late age. Children, on the other hand, because of altruism (or, under an alternative interpretation, because of a contract which is binding within the family), transfer resources to the old parents (possibly in exchange for the inheritance of the family land). How much is transferred is determined endogenously in accordance with different forms of interaction within the family. We look at two sets of social norms which may regulate such interactions. According to the first norm, siblings choose donations to maximize the sum of their utilities. We call this behavior cooperative. We call noncooperative the case in which each child maximizes his own utility, taking the behavior of the other siblings as given.

The exogenous driving force behind the model's dynamics is the infant mortality rate. (Here infant has to be interpreted extensively, as each of our model's three periods of life lasts for 20 years.) As the probability of a child's survival until age 20 changes, parents adjust their fertility decisions and, correspondingly, their land/capital accumulation decisions.

In general, when survival rates are low, fertility is high, but since children are a fairly expensive way of saving for old-age consumption, capital/labor intensity is high. As survival rates increase, fertility decreases, population increases, and the capital/labor ratio decreases. These qualitative predictions hold for both the cooperative and noncooperative versions of the model.

Of particular interest is the prediction that fertility falls as infant survival

rates increase. This is in agreement with the evidence, both time series and cross-sectional, and hence represents an improvement over simple versions of the Barro and Becker model. Moreover, simple calibrated examples of the noncooperative version of the model have the property that the population growth rate is roughly independent of the infant mortality rate. This is in contrast to both the cooperative version of the model and the Barro and Becker model, which both generate large increases in population growth across steady states when the IMR is lowered significantly.

Although the qualitative properties of the cooperative and noncooperative versions of the model are similar, the two versions are strikingly different quantitatively. For example, given parameter values such that the cooperative version of the model generates historically reasonable fertility and population growth rates, the noncooperative version predicts fertility so low that population shrinks. This is because, other things equal, the donations parents can expect from their children are much lower under noncooperative behavior, giving rise to much lower fertility levels. On the other hand, when each model is calibrated to match the population growth rates of England in the 1600s, the size of the reduction in fertility in the steady state from a given reduction in the infant mortality rate is much the noncooperative version of the model. Indeed, in the noncooperative version, given historical levels of infant mortality rates for both the 1600s and the 1990s, the overall population growth rate is unchanged across steady states.

The model also seems to be able to replicate other important features of the historical data. In particular, the model captures well the delayed fertility response to either a plague or a famine, the parallel increase in the real wage, and the subsequent return of population, per capita productivity, and fertility to their initial levels after the shock has passed.

Where our current framework fails most dramatically is in modeling the hump-shaped, or nonmonotone, behavior of the growth rate of the population as the demographic transition unravels. As shown, fertility rates decrease monotonically but never enough to eventually reduce the population growth rate. In the data, a reduction in population growth rates sets in (quite rapidly) after the period of high growth. European and North American evidence suggests that zero population growth may be the final outcome of

the demographic transition. We are not aware of any intertemporal economic model that is based on microfoundations and is capable of mimicking this last stage together with the first two. In fact, we are not aware of any model which can reasonably replicate the quantitative dynamic features of even the first two stages.

The model presented here replicates the first two stages reasonably well. In future research, we plan to study the effect of lowering over time the percentage of the population behaving cooperatively and increasing the percentage behaving noncooperatively as the IMR evolves according to its historical pattern. This switch from cooperative to noncooperative behavior would follow the pattern of urbanization that is seen in the historical record. The idea here is that family ties are weakened when the population moves from agriculture to manufacturing. If the basic quantitative properties of the models that we have uncovered here continue to hold in this more complex environment, this transition could successfully mimic the third stage of the demographic transition as well. We also plan to study the quantitative impacts on fertility of the increase in social security systems in developed countries in recent years and address the question of the extent to which these impacts can be used to explain the recent reductions in fertility in Europe.

An important task for our future research is to show that our conjecture holds in a full-blown dynamic model of fertility and capital accumulation in which social norms and mortality rates are both changing over time.

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