

# THREE EQUATIONS GENERATING AN INDUSTRIAL REVOLUTION?

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**ABSTRACT.** In this paper we evaluate quantitatively the relationship between economic and demographic growth. We use simple models of endogenous growth, featuring human capital investment at the individual level in conjunction with either of two models of fertility choice, the B&B dynastic motive, and the B&J late-age-security motive. We find that exogenous improvements in age-specific survival probabilities, lead to increases in the rate of return to human capital investment. This, in turn, engenders an increase in the growth rate of output per capita much like what is typically seen when countries go through industrialization. In the models implemented so far, we also find that this is accompanied either by a permanent increase in the growth rate of population (for the B&B), or by too much late age consumption and a rate of return on capital that is too high relative to available data (for the B&J). Historical records show that the increase in the growth rate of population during the takeoff was temporary, and that the transition eventually lead to a stationary population; it is unclear if a different version of the B&B model can be built that generates this crucial stylized facts. On the other hand, preliminary work shows that more realistic variations of the B&J model should eliminate the unrealistic predictions about old age consumption and rate of return on capital while maintaining the ability of the model to generate an IR and a DT.

## 1. INTRODUCTION

The idea we pursue in this paper is simple, still ambitious. We lay out a simple model (in fact: two family of models) of fertility choice, production, and capital accumulation in which an increase in life expectancy may, in principle, cause a sustained increase in the growth rates of productivity and income per-capita. We use different variants of these models to assess the quantitative relevance of a contention that has frequently been advanced in the literature on economic growth, and in the context of studies of the British Industrial Revolution especially. It can be summarized as follows: exogenous improvements in survival probabilities provide the incentive for investing more in human capital

and productive skills, hence leading to an increase in the productivity of labor; the latter, through some “endogenous growth” mechanism, leads to a sustained increase in the growth rate of output. In other words, the productivity take-off that characterizes most development processes may be mostly due to exogenous improvements in the life expectancy of the population. Furthermore, while the initial increase in lifetime expectancy brings about also an increase in population and, possibly, of its growth rate, both the increase in the productivity of (parental) labor time and the enhanced incentives to invest in human capital (of children) are likely to activate a quantity-quality tradeoff that leads to a reduction in fertility and, eventually, to a newly stagnant population.

The intuition is straightforward, and it may actually work. Indeed, it does seem to work in a variety of different theoretical formulations of the basic intuition, as an extensive literature (discussed in Section 3) has shown in the recent past. The intuition may also work in practice, i.e., once solid historical evidence has been used to discipline calibration, a fully specified model of endogenous growth and fertility may deliver (most of) an industrial revolution and a demographic transition using the historically observed changes in (young’s and adult’s) life expectancies as the only exogenous “causes”. In fact, a number of papers have compared numerical simulations of simplified models with data and claimed that the ‘basic’ stylized facts of economic takeoffs and demographic transitions are replicated. The most relevant among these attempts we also discuss in Section 3; in our judgement, things are less clear cut than the literature so far seems to believe and the jury is still out on the overall capability of a well specified model of the ‘survival hypothesis’ to generate an industrial revolution *and* a demographic transition. As we argue in Section 3 on the basis of the historical evidence summarized in Section 2, it remains an open question if using historically observed time series of (age specific) survival probabilities, and realistically calibrated technologies, one can actually generate time series of output, capital stock, wages, rental rates, and fertility that resemble those observed in UK during the 1700-2000 time interval. This paper aims straight at this question, and the *preliminary* answer is a qualified ‘maybe’, leaning more toward yes than no. Lots of non-trivial work still needs to be carried out, by us and by other, to reach a convincing answer.

In spite of its current appearances, our paper is not a theoretical one; this research is meant, instead, to be an exercise in quantitative economic history. For this reason, we do not claim that our model is able to capture “all” industrial revolution-like sustained increases in

the growth rate of income per-capita. We concern ourselves with a specific historical episode: the English Industrial Revolution of the eighteenth and nineteenth centuries, and use more or less contemporaneous episodes in other European countries purely to test the robustness of certain stylized facts. This is not to rule out beforehand that the causal explanation examined here may turn out to have “universal” value. In fact, we suspect it may, in so far as the majority of eighteenth and nineteenth century take-off episodes we are aware of, were preceded by substantial changes in the patterns of mortality, and by a more or less visible increases in life expectancy. Before drawing such conclusion, though, the ability of a differently calibrated version of this model to replicate other episodes of economic and demographic growth should be assessed. What we have learned so far is that, when calibrated to the data for England 1700-2000, our two models exhibit an endogenous change in the rates of physical and human capital accumulation, fertility, and labor productivity which is similar (but not of the same magnitude of) those observed in the data.

Before moving on to discuss motivational evidence and previous literature, we should indulge a bit longer in a discussion of the basic mechanism inspected in this paper, and of the modeling choices we have made. Among the many differences between physical and human capital, one is attributable to human mortality: when people die, their human capital ( $h$ ) goes away with them, while their physical capital ( $k$ ) is left behind to their descendants. Hence, everything else the same, the  $k/h$  ratio an individual acquires is decreasing in the length of time one expect to be able to use  $h$ . An increase, at the time of human capital investment decisions, in the expected future flow of workable hours implies that the rate of return on human capital investment goes up. If the underlying production function has the form  $F(k, h)$ , an increase in  $h/k$  increases the return on  $k$  in turn, thereby increasing the overall productivity of the economy. This change causes the growth rates of output per capita to increase in models where output growth is endogenous and determined by equilibrium conditions of the type: growth rate of consumption equal to a monotone increasing transformation of the discount factor times the marginal productivity of capital. Hence, our choice of a linear homogeneous production function, in which output per capita  $y = F(k, h)$ , and the two inputs can be reproduced and accumulated without bound from one period to the next. This choice of technology rules out changes in the relative price of  $h$  and  $k$ , something that may well have taken place but which does not seem to follow, either directly or indirectly, from the ‘survival hypothesis’. We have also chosen to abstract from ‘exogenous’ TFP in our model, be it neutral

or, as in other exercises, ‘biased’ toward skilled labor. This is only for a matter of simplicity and not because we believe that changes in per capita human capital alone can possibly explain the whole evolution of U.K. aggregate productivity from 1700 to 2000. In fact, as Section 2 acknowledges in reviewing the cumulated historical evidence on the sources of productivity growth, this cannot clearly be the case, and a more complete account of the British Industrial Revolution should also model and quantify the key technological changes that took place during those two centuries.

In order to work, the causal mechanism described here needs to satisfy an additional couple of requisites. First, a generic increase in life expectancy would not do, as the latter can be determined, for example, by a drop in infant or child mortality or by an increase in late age survival: in both cases the amount of time during which productive human capital can be used would not change. The survival rate that matters is the one between the age at which productive skills are acquired and the age of retirement from the work force, that is between 10-15 and 50-60 years of age two centuries ago, and between 20-25 and 60-65 years of age nowadays. This explains why we have paid a substantial degree of attention to the life cycle and the timing of human capital acquisition/utilization. Second, if the survival mechanism has to work, the change in survival rates must eventually induce a reduction in total fertility, that is they must be correlated to a movement along the quantity/quality dimension among offsprings. Ideally, what one would like to have is that births fall, survivors increase, investment per survivor rises, time spent by survivors in productive activities increase, hence incentive to invest in both  $h$  and  $k$  also increase. A sharp enough reduction in birth rates is not an obvious consequence of an increase in child and youth survival probabilities, as the ‘higher return on children’ the latter implies may, in reasonable circumstances, lead to an increase in the number of children; something that is well known to happen, at realistic parameter values, in models of endogenous fertility. This is a crucial feature of the problem at hand, which has lead us to inspect two different (almost opposite) rationalizations of why people have children and why they may or may not invest in their training. The first is the well known ‘dynastic altruism’ motivation of Becker and Barro (1988), while the second is commonly known as the ‘late-age-insurance’ motive, and we use the formalization advanced in Boldrin and Jones (2002). There is, therefore, a sense in which part of our paper consists of a “horse race” between different views of endogenous fertility. As mentioned, we know from previous work (i.e. Boldrin and Jones (2002)) that the B&B explanation tends to perform badly when used

to model endogenous fertility reactions to changes in mortality rates in models of exogenous growth. We are interested to see how it does when the same changes in mortality rates are used to generate changes in human capital accumulation over the life cycle and increases in the growth rate of output and labor productivity. Also, we know that the B&J explanation, in its non-cooperative version,<sup>1</sup> tends to predict levels of fertility that are low by historical standards, and much more elastic (downward) to reductions in mortality rates than the cooperative version, which in turn predicts higher fertility rates. Again, such features of the B&J model appear, at realistic parameter values, when growth is driven by exogenous TFP, and how performances change in an endogenous growth context is neither clear nor obvious.<sup>2</sup>

Additionally, as we like to impose upon our quantitative exercise the discipline of historical measurement, we do abstain from introducing into our models a number of ‘external effects’ that are frequently encountered in this literature, and that often become the main driving force behind the final results. This is not meant to imply we are completely convinced that a number of external effects, especially when it comes to the impact of public health and education, cannot have played a role in the evolution of mortality rates and overall productivity during the British Industrial Revolution. In principle, they may have been quite relevant, but no quantitative measurement of their nature, size and impact is anywhere to be found. In this sense, ‘human capital externalities’ are a bit like *phlogiston*, a fascinating and theoretically useful, but hardly measurable entity. Our choice of sticking to models where external effects are altogether absent means, then, that at least when trying to perform an exercise in quantitative economic history, we deem it appropriate to adhere to the rule ‘whereof one cannot measure, thereof one should not assume.’

## 2. MOTIVATIONAL AND HISTORICAL EVIDENCE

The idea that there is a connection between economic development and population dynamics is an old one. It appears in many forms in the literature in both areas. Indeed much of the literature on the Demographic Transition has focused on this link as a causal one –

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<sup>1</sup>We apologize to the reader for the, temporarily, cryptic language, that should become less so once the formal models are introduced. The details of the cooperative and non-cooperative case are discussed in Boldrin and Jones (2002).

<sup>2</sup>In this preliminary version of the paper, it remains unclear what the answer to this question is. Much to our chagrin, we are still far from satisfactorily calibrating and simulating the B&J model for versions other than the simplest one, and even then for BGPs only.

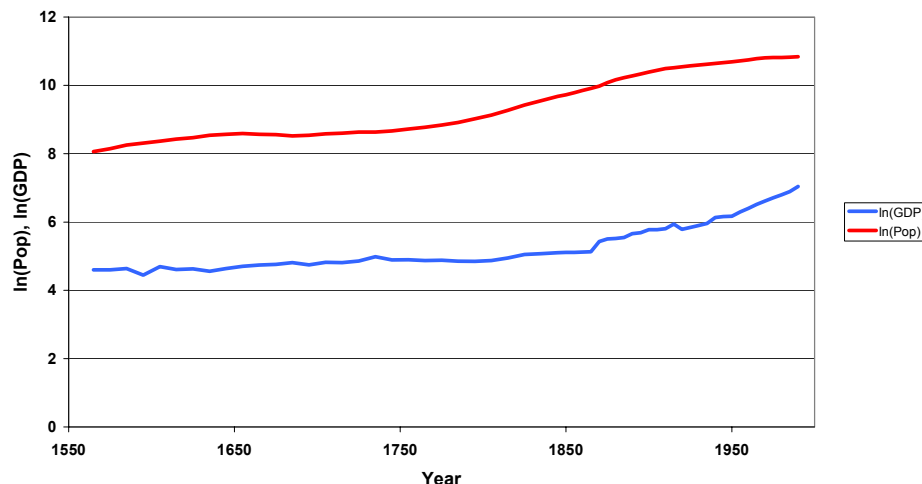
**Figure 1A: UK Population and GDP per capita, 1565 to 1990**

FIGURE 2.1

the reduction in fertility is caused by the increase in per capita GDP that goes along with the development process. The reasoning that the relationship is causal is tenuous at best and really comes down to the observation that the two series are co-integrated. Figure 1A below shows the basics of the relationship in a historical setting in the UK for the period 1565-1990; data are reported in logs to facilitate the visual capture of changes in the underlying trends.<sup>3</sup>

For output, it can be seen that there is a slow but steady climb over the period from 1550 to about 1830 (or 1850). At that point output growth accelerates as indicated by the change of slope in the series. Population also shows a slow steady climb at first, lasting until about 1750. After that point, there are two distinct changes. First is the acceleration that occurred between 1750 and about 2000, and then the dramatic slowdown after that, with the series becoming almost flat since 1960 or so.

Figure 1B tells a similar story, but in growth rates rather than log-levels. These series (naturally) show much more volatility than those

<sup>3</sup>Data sources are as follows. Population data, 1541-1866: Wrigley, Davies, Oepen and Schofield, "English population history from family reconstitution"; 1871-1970: Mitchell, "European Historical Statistics". GDP data are taken from Michael Bar and Oksana Leukhina (2005). The basic sources are Clark from 1565 to 1865, and Maddison from 1820 to 1990. Clark is normalized so that 1565 = 100, Maddison is renormalized so that Maddison = Clark in 1865.

Figure 1B: GDP and Pop Growth Rates: UK 1565 to 1990

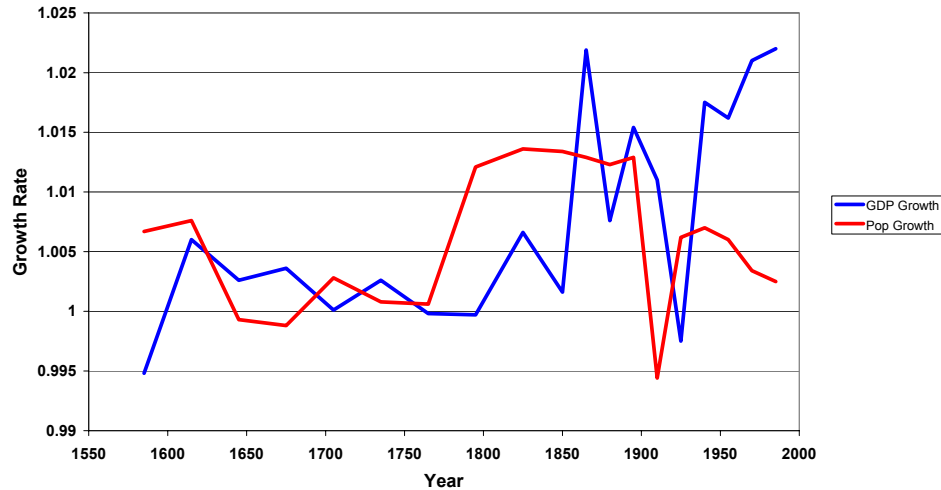


FIGURE 2.2

in Figure 1A, where the visual image is swamped by growth. Even still one can easily see the changes discussed above as the average growth rate of output shifts clearly upward (albeit volatily so) and the growth rate of population first goes up, then later, around 1900, falls again. An important additional feature of the UK data, which is only weakly replicated in data from other European countries (see Appendix II) is the following: around 1770 the population growth rate increases substantially, and remains high for about a century and a half, before dropping to levels near zero. This increase in population growth anticipates the increase in output growth of between 50 and a 100 years<sup>4</sup>, but output growth remains high after that or even increases till our days.<sup>5</sup> These two differences are crucial for our testing of the ‘survival hypothesis’: (i) population growth increases *before* income growth does, (ii) the increase in population growth rate is *temporary*, and not accelerating, while that in per capita GDP is *permanent*, and accelerating for about two centuries.

<sup>4</sup>For more details about output growth between 1700 and 1900, and on the modifications of historical estimates brought about by recent work, see later in this Section.

<sup>5</sup>Clearly, we are abstracting from the impact of the Great Depression, which is visible in Figure 1B.

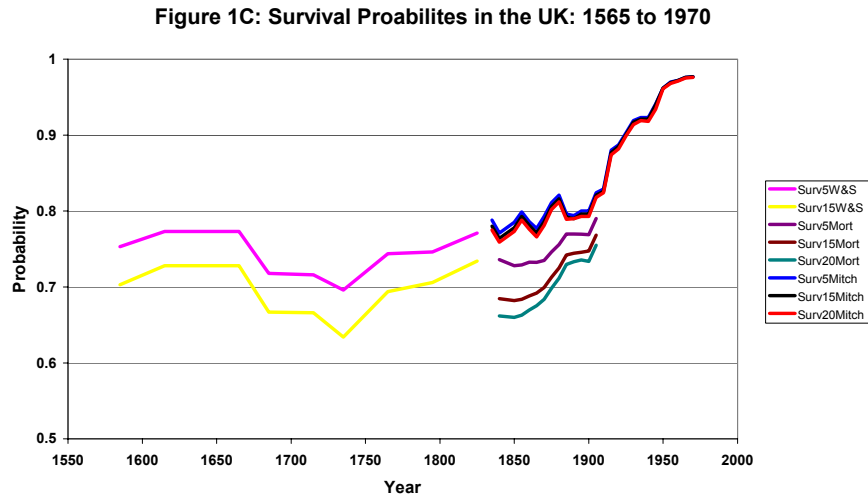


FIGURE 2.3

Since this is of interest for our modeling choices, and relevant to assess the ability of different models to replicate the actual causal mechanism, if any, between survival probabilities and economic growth, we also include here the historical record on survival probabilities at different ages.<sup>6</sup> Shown in Figure 1C are time series of the probability of surviving to ages 5, 15 and 20 since 1565. As can be seen survival rates to working age have shown a dramatic increase in 1880 or so. But also note that the survival rate to age 15 may have picked up substantially earlier, around 1750 or so.

[SURVIVAL PROBABILITIES TO 40 AND 60 YEARS TO BE ADDED]

The same kind of relationship IS seen in cross sectional data in more modern times. For example, cross sectional data on the average, over the 1960 to 2000 period, rate of growth of per capita GDP and the average, again over the 1960 to 2000 period, rate of growth of population for a sample of 98 countries taken from the Penn World Tables is shown in Figure 2.

<sup>6</sup>Survival Data sources are, 1541-1866: Wrigley et al. (); 1840-1906: <http://www.mortality.org>; 1906-1990: Mitchell ().



Figure 2: Real GDP per capita Growth - Population Growth

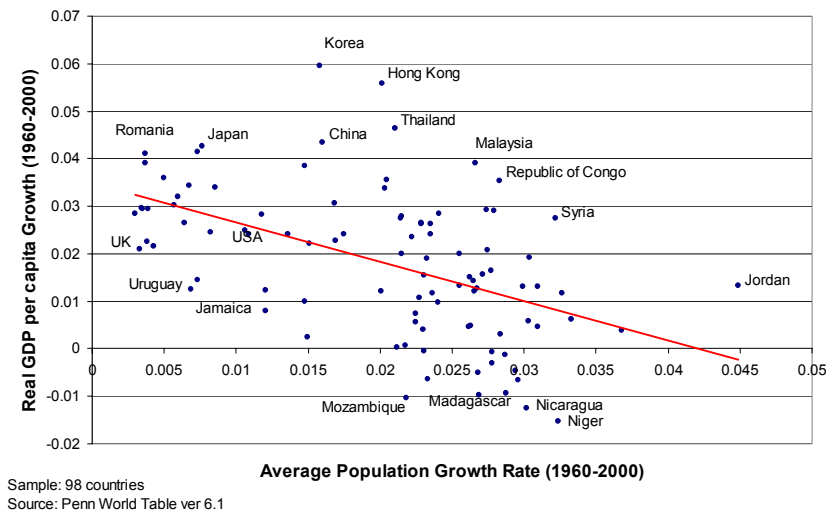


FIGURE 2.4

The relationship between  $\gamma_y$  (per capita GDP growth) and  $\gamma_n$  (population growth) is statistically and economically significant. Statistically, the estimated relationship is:

$$\gamma_y = 0.0343 - 0.826\gamma_n$$

The  $R^2$  of this regression is 25.4%, and the standard error of the regression coefficient is 0.1444, giving a t-ratio of -5.72. Thus, with every decrease of 1% in population growth rates, there is an associated 0.83% increase in per capita output growth rates, economically a very large change. The strong correlation evidenced here is purely motivational, and we should focus next on more detailed UK data for the period 1700-2000. Still, the reader should also consult () and () for further analysis of recent cross country data and the claim that the causal mechanism investigated here is actually relevant to understand current development experiences.

The historical evidence we consider next is of three types: (i) estimates of the historical evolution of per capita GDP in the U.K. from 1700 to the early 1900s (DATA FOR XX CENTURY TO BE ADDED); (ii) data on the demographic transition in the U.K. during the same period, with a particular attention to the evolution of age-specific mortality rates and fertility; (iii) additional evidence, both from the U.K. and other countries, of the timing of demographic and economic change.

**2.1. Output Growth and its Sources.** The following table is constructed using estimates reported in Crafts (1995), and Crafts and Harley (1992). The factor shares used to compute the estimates are, respectively,  $K = 0.40$ ,  $L = 0.35$  and  $H = 0.25$ , where  $K$  is the aggregate capital stock,  $H$  is a measure of human capital<sup>7</sup> and  $L$  is total raw labor. While this parameterization of the aggregate technology is not exactly equivalent to the one we use in our modeling and calibration exercise, it is the closest available in the literature, and should provide a reasonable ‘target’ for the kind of detailed facts the formal model should be able to match in order to be called ‘successful’.

**Table 1: Output Growth, 1700-1913 (% per year)**

| Period    | $\Delta Y/Y$ | $\Delta K/K$ | $\Delta L/L$ | $\Delta H/H$ | $\Delta TFP$ |
|-----------|--------------|--------------|--------------|--------------|--------------|
| 1700-1760 | 0.7          | -            | -            | -            | -            |
| 1760-1780 | 0.6          | 0.25         | 0.20         | 0.10         | 0.05         |
| 1780-1801 | 1.4          | 0.45         | 0.35         | 0.40         | 0.10         |
| 1801-1831 | 1.9          | 0.70         | 0.40         | 0.55         | 0.25         |
| 1831-1873 | 2.4          | 0.90         | 0.45         | 0.70         | 0.35         |
| 1873-1899 | 2.1          | 0.80         | 0.30         | 0.50         | 0.50         |
| 1899-1913 | 1.4          | 0.80         | 0.30         | 0.50         | -0.20        |

The most striking feature of Table 1 is the very small growth rate of TFP all along the two centuries under consideration, and especially until 1850. This finding, that aggregate TFP growth was quite small during the initial two centuries of the British Industrial Revolution, and that accumulation of factors was the main driving force, characterized Crafts and Harley fundamental contribution to the economic history literature, and has helped to substantially alter the view of most economic historians about the nature of the Industrial Revolution. When first published, the Crafts and Harley estimates of output and productivity growth were received with doubts, and gave rise to a fairly intense debate. Their basic contention was that substantial technological improvements did take place during the period, some of which did imply revolutionary changes, but such improvements were restricted to very few sectors of the UK economy. More precisely, their historical work shows that dramatic increases in productivity took place in the textile (especially cotton), shipping, and metallurgical industries, the size of which were, nevertheless, too small to generate the increase in aggregate TFP that previous researchers has estimated. Our readings of the literature that ensued, and of the current consensus among economic historians, is that the Crafts and Harley estimates, including minor

<sup>7</sup>Add details on how this measure is constructed.

recent revisions, are the best available and, short of new additional evidence, should be taken as summarizing the basic facts any model of the British Industrial Revolution should be capable of matching. This is, at least, the position adopted in this paper. In summary, from Table 1 we learn two things our modeling exercise must be able to account for:

- (1) Output growth did not increase suddenly from the previous 0.3 – 0.5% per year rate<sup>8</sup> to 2% a year. The increase was slow and progressive, and even more so (see below Table ??) if one consider output per employee or output per capita (recall from above that the population growth rate is close to 1.5% per year between 1770 and 1890.)
- (2) The contribution of physical and human capital accumulation to output growth was quantitatively more important than that of TFP during the whole period. It is only at the very end of the nineteenth century that TFP growth starts to become substantial.

Table 2, also based on Crafts and Harley estimates as reported in Clark (2004, Chapt. 9), confirms and qualify these findings by using an aggregate production function that excludes direct measures of human capital but introduces land ( $Q$ ). Again, TFP growth is slow at the beginning and progressively accelerates, and, again, a sharp increase in capital to labor intensity accounts for most of the growth in labor productivity until the second half of the nineteenth century. In these estimates, the factor shares adopted were of 35% for capital, 15% for land and 50% for labor. For detailed estimates of factors share, see Stokey (2001:69), showing that labor share goes from .45 to .47 between 1780 and 1850, capital share goes from .35 to .44 and land share goes from .20 to .09 during the same period.

**Table 2: Per Capita Productivity Growth (% per year)**

| Period    | $Y/L$ | $K/L$ | $Q/L$ | $TFP$ |
|-----------|-------|-------|-------|-------|
| 1700-1760 | 0.30  | 0.31  | -0.29 | 0.24  |
| 1801-1831 | 0.45  | 0.36  | -1.35 | 0.53  |
| 1831-1861 | 1.09  | 0.59  | -1.42 | 1.10  |
| 1760-1861 | 0.54  | 0.37  | -1.12 | 0.58  |

Finally Table 3, adapted from Clark (2001, 2004) reports a number of other measures of factor prices and productive efficiency for the period of our interest. Wages are computed for a 10 hour day, land

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<sup>8</sup>For the most recent and reliable estimates of the growth rate of output pre-1700 in the UK see Clark (200?) and Feinstein (199?).

rent is in £ per acre, and the return on capital is in percentage points. The index of productive efficiency is normalized to a value of 100 for the decade 1860-1869. It provides a further confirmation of what we already said: TFP growth was slow at the beginning, accelerating only well into the nineteenth century. Notice also that, while the return on capital is quite stable, the return on land increases dramatically, as does the wage rate per day of work after 1800.

**Table 3: Efficiency Growth in the Industrial Revolution**

| Decade | W    | R on Q | R on K | Input Costs | Efficiency |
|--------|------|--------|--------|-------------|------------|
| 1700-9 | 11.7 | 0.45   | 4.67   | 60.7        | -          |
| 1710-9 | 12.1 | 0.48   | 4.96   | 60.9        |            |
| 1720-9 | 12.3 | 0.52   | 4.38   | 62.9        |            |
| 1730-9 | 12.7 | 0.51   | 4.14   | 69.7        | 68.9       |
| 1740-9 | 12.9 | 0.47   | 4.24   | 71.9        | 64.8       |
| 1750-9 | 12.9 | 0.59   | 4.26   | 78.5        | 65.6       |
| 1760-9 | 13.7 | 0.59   | 4.04   | 87.5        | 64.4       |
| 1770-9 | 14.6 | 0.69   | 4.15   | 102.2       | 64.4       |
| 1780-9 | 15.3 | 0.68   | 3.95   | 112.1       | 64.3       |
| 1790-9 | 18.2 | 0.83   | 4.10   | 142.9       | 66.3       |
| 1800-9 | 25.1 | 1.15   | 4.38   | 215.5       | 70.4       |
| 1810-9 | 34.1 | 1.47   | 4.63   | 296.5       | 81.6       |
| 1820-9 | 31.3 | 1.29   | 4.48   | 309.4       | 88.3       |
| 1930-9 | 32.8 | 1.22   | 4.85   | 341.5       | 95.0       |
| 1840-9 | 33.6 | 1.15   | 4.28   | 398.0       | 97.4       |
| 1850-9 | 35.4 | 1.24   | 4.10   | 462.8       | 98.9       |
| 1860-9 | 39.8 | 1.37   | 4.27   | 581.4       | 100.00     |

Clark (2002), also shows that agricultural wages decline from 1540 all the way till 1640, when a slow recovery begins that is reversed again about a century later. It is only by the middle of the 19th century that real agricultural wages go back to their level of 300 years before, and grow monotonically to new heights every period after that. The indices he computes for the productivity of capital in the agricultural sector show an almost parallel dynamics, even if the levels are different and, in particular, the capital productivity level of 1550 is reached and surpassed many times between 1620 and 1850; nevertheless, it is not until 1800 that a monotone increasing trend sets in. Interestingly, the estimated rates of return decrease from historical highs of 6.2 % in 1600-1620 to 4.2% in 1800 and then to 3.9% in 1900.

Various other data sources suggest that accumulation of productive factors played a major role in output growth, at least until 1850 and

probably after that. For example, the share of consumption over income decreased from 93% in 1700 to 80% in 1840, while the investment to output ratio more than doubled, from 4.0% to almost 11% in 1840 (Feinstein 1988). Population and net fixed capital formation growth rates are similar and rising, period after period, between 1760 and about 1810. After that the population growth rate decreases, from a pick of about 1.4%, while capital formation growth rate accelerates and reaches almost 3% by 1840. (Nick von Tunzelmann, in Floud and McCloskey, eds. (1994), p. 290). What this implies is that the K/L ratio increases, slowly but steadily, from the middle of the eighteenth century onward, and that after 1810 or so its growth rate accelerates substantially.

MORE ON MEASURES OF LABOR PRODUCTIVITY AND FACTOR SHARES TO BE ADDED

**2.2. Mortality and Fertility.** What happened, during the same period, to mortality and fertility rates? Figure 3 summarizes the answer: (a) mortality rates remained high well into the eighteenth century, improving only marginally toward its end, and then only for adult people; (b) fertility tagged along with mortality rate as before, but then took off substantially in the second half of the eighteenth century, leading to a dramatic increase in the growth rate of the population; (c) infant (and child) mortality did not improve seriously only until well into the nineteenth century, and it is only in the very second part of this century that dramatic, and dramatically fast, improvements take place.

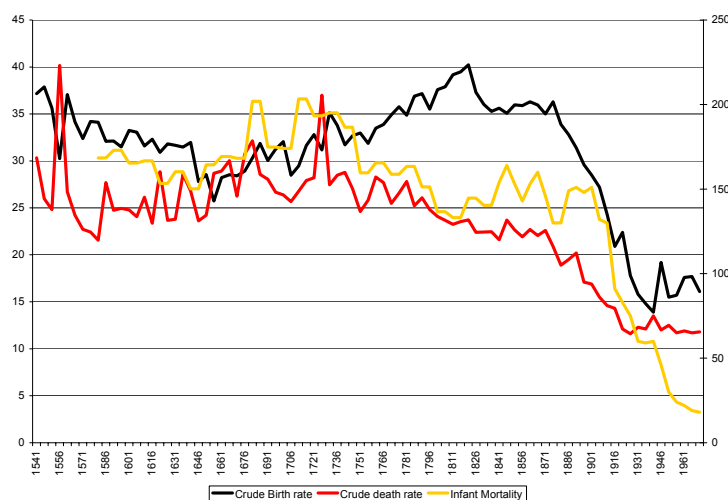


FIGURE 2.5. **Figure 3: British Crude Birth, Death and Infant Mortality Rates 1541-1970**

These mortality and fertility patterns generate the compounded population annual growth rates reported in Table 4 (from Wrigley and Schofield (1981: 208-9)), and the Gross Reproduction Rates<sup>9</sup> and expectation of life at birth reported in Table 5 (also, adapted from Wrigley and Schofield (1981:231)). Both ventennial and decennial values have been obtained by averaging over the quinquennial values that Wrigley and Schofield report. There is one important message one wants to learn here: life expectancy did not increase by any substantial amount between 1700 and 1870, and this is because infant and child mortalities

<sup>9</sup>The GRR is, basically, the same as the TFR restricted to female offsprings. It does, therefore, take into account the fact that male infants are slightly more likely than female infants.

did remain very high until late in the eighteenth century. On the other hand, as Figure 1C above shows, survival probabilities from age five to age twenty did increase of a visible amount during the same period. It is with this quantitative restrictions on the actual improvements in mortality rates that a model testing the 'survival hypothesis' for the British Industrial Revolution has to cope. Life expectancy is not only the inappropriate theoretical measure to look at, but, historically, it started to grow only when the IR was well under way; expectation of life at birth, between 1681 and 1741 oscillates from a minimum of 27.9 years to a maximum of 37.1 years, from there it slowly raises to reach 40 years in 1826-1831 and 41.3 in 1871.

**Table 4: English Population compound annual growth rates**  
1681-1701 1701-1721 1721-1741 1741-1761 1761-781

|           |           |           |           |           |
|-----------|-----------|-----------|-----------|-----------|
| 0.25      | 0.28      | 0.21      | 0.49      | 0.68      |
| 1781-1801 | 1801-1821 | 1821-1841 | 1841-1861 | 1861-1881 |
| 1.04      | 1.42      | 1.08      | 1.18      | 1.26      |

**Table 5: GRR and Life Expectancy**

|         |      |      |
|---------|------|------|
| 1681-91 | 2.06 | 30.1 |
| 1691-01 | 2.17 | 34.5 |
| 1701-11 | 2.29 | 36.8 |
| 1711-21 | 2.15 | 36.5 |
| 1721-31 | 2.24 | 32.5 |
| 1731-41 | 2.28 | 32.3 |
| 1741-51 | 2.25 | 33.5 |
| 1751-61 | 2.33 | 37.0 |
| 1761-71 | 2.38 | 34.6 |
| 1771-81 | 2.51 | 36.9 |
| 1781-91 | 2.56 | 35.3 |
| 1791-01 | 2.76 | 37.0 |
| 1801-11 | 2.81 | 37.3 |
| 1811-21 | 2.97 | 37.8 |
| 1821-31 | 2.92 | 39.6 |
| 1831-41 | 2.56 | 40.5 |
| 1841-51 | 2.43 | 39.9 |
| 1851-61 | 2.42 | 40.0 |
| 1861-71 | 2.53 | 40.7 |

Our allegation, TO BE BETTER DOCUMENTED, is that such increase in survival rates, first slow and limited to the adult segments

of the population and then much more dramatic, generalized and particularly strong among children and infant, is to be considered as an *exogenous technological (or medical) shock* and not as the effect of the improvement in the living conditions of the British people that took place before it. The fundamental motivation for our position is that, on the one hand, an overwhelming number of demographic studies (see Livi Bacci (2000) for an excellent summary) have documented that the range of caloric-proteinic intake and living conditions compatible with human survival is so large that the purely economic improvement in the living conditions of the British people that took place between 1700 and 1800 cannot possibly be the cause for the increase in survival rates that ensued. On the other hand, our reading of the specific causes of the TO BE ADDED The late 1720s is the last time in which England is affected by epidemics in any relevant form; in 1731 the total population is at 5.263 million. After that, the population growth rate starts accelerating, peaking at 1.55% per year in the quinquennium 1826-1831 and then falling to around 1.2% a year until 1871, at which point the population of England had quadrupled to 21.501 million.

Razzell (1993) contains a strong advocacy of the idea that “Population growth in 18th century England was due mainly to a fall in mortality, which was particularly marked during the first half of the century. The fall affected all socioeconomic groups and does not appear to have occurred for primarily economic reasons.”

The idea that is income growth that causes a decline in mortality, and not viceversa, finds strong support, at least *prima face*, from the observation that it is only in the second half of the 19th century that life expectancy increases substantially, and this is at least a century after income per capita had started to visibly increase in Europe, and in the U.K. in particular. Further, this is not true only for the U.K. but seems to be a general feature of most countries: for all European countries life expectancy trend changes between one and half a century after per capita GDP trend changes, and the growth rate in life expectancy after the take off is three to six times higher than before; see, e.g. Easterlin (1999, Tables 1 and 2). One should hasten to add, though, that looking only at life expectancy at birth is rather distorting, and that the sharp increase

Further, the same data also show that this fact is not true for late growing countries such as Brazil or India: in their case, life expectancy and economic growth move either simultaneously or the former earlier than the latter, suggesting that life expectancy is mostly driven by the spread of advanced medical practices, and that the latter, while vaguely



correlated with income levels, tend to follow a dynamics that is mostly independent from that of per capita income growth.

TO BE ADDED

### 3. REVIEW OF PREVIOUS LITERATURE

Libraries have been written, both on the Industrial Revolution and on the Demographic Transition in the United Kingdom between 1600 and 2000, hence attempting a true review of the literature would result in an impossible task. We focus here only on the recent literature, say post 1980, and, within this, on three kinds of works: (i) those providing data and empirical support to the particular causation mechanism we are testing; (ii) those modeling such causation mechanism, in particular those works that have attempted to bridge the gap between theory and data; (iii) those works that explicitly question the direction of causality we are concerned with and suggest that either the opposite is true or that changes in age-specific survival probabilities had little or nothing to do with the increase in the growth rate of output per capita in the U.K after 1750.

One of the first researchers to begin addressing the potential causal link between health and economic growth is Samuel Preston, see Preston (1975). His conclusion, based on data for the period 1900-1960, was that health improvements were mostly due to autonomous technological change in the ‘health production function’, with little contribution from increases in income per capita per-se. On the opposite side, and practically at the same time, Thomas McKeown, McKeown (1976), uses data for England and Wales from the middle of the 19th century to argue that it could not possibly be the case that improvements in medical technology were capable of explaining the increase in life expectancy observed at the time in those areas.

McKeown’s argument, which was interpreted mostly as an argument for the market as a provider of health services, has been widely criticized ever since, see Mercer (1990) and Easterly (1999) for recent examples and review of other works. This criticism TO BE ADDED

Among the most interesting recent economic papers that investigate the same causation channel as we do, the following are closer to our approach: Boucekkine, de la Croix and Licandro (2002, 2003), Cervellati and Sunde (2002), Kalemli-Ozcam (2002, 2003), Soares (2005), and Zhang, Zhang and Lee (2001). DISCUSSION TO BE ADDED.

Hazan and Zoabi contains a sharp theoretical criticism to most of the modeling strategies adopted in the previous literature. Correctly, we believe, they point out that “[...] greater longevity raises children’s

future income proportionally at all levels of education, leaving the relative return between quality and quantity unaffected. [...] Our theory also casts doubts on recent findings about a positive effect of health on education. This is because health raises the marginal return on quality and quantity, resulting in an ambiguous effect on the accumulation of human capital.” This criticism does not apply to the models studied here, as it is the incentive to invest in physical versus human capital that is affected by the changes in survival probabilities. Further, we investigate age specific survival probabilities directly, and allow for the quantity/quality tradeoff to be resolved at the quantitative level.

In most of the literature on demographic behavior and economic growth, the problem is almost invariably cast in the following very stylized form: before the second half of the 18th century European countries, and the rest of the world indeed, were at a stationary Malthusian equilibrium with low (approximated to zero) growth in income per capita, high mortality, high fertility, low (approximated to zero) growth of population and small or nihil investment in human capital and productive skills. A transition to a new stationary state of sustained growth in income per capita, low fertility and mortality (in fact, constant population) and high investment in productive human capital begins around 1750 and is completed roughly two century later. During the transition, mortality and fertility decline somewhat in parallel, human capital accumulation progresses and the growth rate of income per capita increases till it stabilizes at around two per cent per year after WWII, finally, population multiplies by a factor of five before becoming constant again. While there is lots of truth in this stylized story, we actually believe it is a *bit too much stylized*. Because it is too much stylized it is consistent with very many different and incompatible explanations, among which it is hard, not to say impossible, to discriminate when the set of “facts” to be accounted by the theory is restricted to those we just listed.

Additionally, a variety of othe authors have investigated in isolation one of more of the mechanisms we assemble here. The one most often encountered is the idea that an increase in life expectancy increases the incentive to acquire human capital (Blackburn and Cipriani [2004]). Cavalcanti and Abreu de Pessoa (2003) look at the interaction between longevity and government fiscal and education policies. TO BE ADDED.

It may not be unfair to say that the “dominant” view is the one summarized in the ‘Malthus to Solow’ parable of Hansen and Prescott (2002) and in the ‘development miracle’ Lucas (2002). While we concur with the general description of the facts presented in these two papers,

and in the many other that follow similar lines, we find the underlying theory lacking in so far as some 'exogenous miracle' becomes the essential driving force, be it exogenous TFP growth or a particularly engineered demand for children or a pervasive and aggregate human capital externality. TO BE ADDED

#### 4. MODELS OF ENDOGENOUS FERTILITY AND GROWTH

We will now present two classes of models of endogenous growth and endogenous fertility: the 'dynastic' model of an infinitely lived family (due originally to Becker and Barro (1988), and the 'late age insurance' model, theorized among other by Caldwell (1978). We label the first the B&B model, and the second the B&J model, as we use the particular formalization of the late age insurance motive introduced by Boldrin and Jones (2002).

**4.1. The Cost of Children Under Certainty Equivalence.** One of the key tricks that we will be exploiting is a way of modeling childcare costs in a certainty equivalent environment. The costs of raising a child are many and complex. Moreover, in an uncertain world, like that in the 18th and 19th centuries in the (currently) developed world, and in much of the developing world still today, the cost of raising a child depends on how long it survives. For example, if schooling is only provided for those children reaching age 5, it is only a cost, ex ante, for those expected to survive. In a certainty equivalent world, this can be introduced into standard models of fertility choice by having different costs for different 'ages' of the kids. The formulation we adopt is based on Schoonbroodt (2004).

The key step amounts to properly defining  $\theta_t$ , the total cost of raising one child. For simplicity, assume that there are 3 stages of life for a child. Roughly, they correspond to 'newborn,' ages 0 to 1, 'child,' ages 1 to 5, and 'school aged,' ages 5 to 15. This is supposed to represent a 'rough' division, finer ones would be possible, and will be considered in future versions of this work. Let  $\pi_{2t}$  and  $\pi_{3t}$  denote the probability of survival to the beginning of the second and third phases of childhood conditional on being born. Thus,  $\pi_{2t} = (1 - IMR_t)$ , and  $\pi_{3t} = (1 - CMR_t)$ . Then, we assume that the (expected) cost of a newborn is given by:

$$\theta_t = [b_1 + \pi_{2t}b_2 + \pi_{3t}b_3] h_t w_t + [\alpha_1 + \pi_{2t}\alpha_2 + \pi_{3t}\alpha_3] c_t^m = b_t h_t w_t + \alpha_t c_t^m$$

where,  $b_i$  is the amount of time required from the parents for a child in phase  $i$ , and  $\alpha_i$  is the amount of physical consumption good, relative to parents consumption, required for a child of phase  $i$ . Thus, in this

formulation,  $b_2$  is only spent on those children that actually survive to phase 2, and so forth. Then, denoting with  $\mu_t = \pi_{3t}$  the probability that a born child survives to the age in which he is educated and/or trained in some productive activity, the total cost of rearing  $n_t$  children is given by:

$$n_t[\theta_t + \mu_t h_{t+1}]$$

Finally, let  $\pi_t$  denote the probability to survival to adulthood. The reason that this is a convenient formulation is that, from the point of view of the differential effects of changing survival rates on the incentives for human capital formation, what we probably want to focus on is the data analog of changes in  $\pi/\pi_3$ , that is, the conditional probability to survival to working age given survival to age 5, since no human capital investment is made for those children that die in infancy.

What we do next is to add this formulation into the B&B and B&J models. Throughout this version, we will assume that  $\alpha_1 = \alpha_2 = \alpha_3 = 0$  for simplicity.

#### 4.2. The Aggregate Production Function.

Let production be defined at the individual, disaggregated level, with  $y_t^i = F(k_t(i), \tilde{h}_t(i))$ , for each member  $i$  of the working age cohort, which is composed of  $N_t$  people (depending on which specific model is being used, this may include people from different generations.) Aggregate output is then

$$Y_t = \int_0^{N_t} F(k_t(i), \tilde{h}_t(i)) di = N_t F(k_t, \tilde{h}_t).$$

**4.3. Different Specifications.** There is a large variety of different specifications of the B&B and the B&J models that one would like to consider, eight of which are reported in Appendix I (to becomes sixteen as the T=4 cases are added!) Some of these versions are implementable in both models, while other are not. Those that are common to both B&B and the B&J fertility theories, are the following: (a) different modeling of the length of time spent working versus the length of time spent in training and the length of time spent in retirement, (b) different specifications for the aggregate production function, (c) different specifications for the market arrangement in the provision of child care services. The different versions that are special only to the B&J model are due to (d) different solution concept for the donations game played among the members of the middle age generation. Let us illustrate the structure of the various versions, starting with point (d).

In the B&J model, when they become middle-age children internalize old parents utility from consumption, and therefore are motivated to provide them with some transfer of consumption goods, which we label “donations” and denote with  $d_t$ . How much per capita donation is chosen, in equilibrium, is nevertheless not uniquely determined by how much children like their parents, which we denote with  $\zeta u(c_t^o)$ , as the within-siblings agreement about how individual donations should be arranged also matters. Two polar cases are relevant: in the *cooperative* one siblings maximize joint utility by choosing a common donation  $d$ ; in the *noncooperative* case, each person take the other siblings’ donation as given and chooses its best response, with the equilibrium being symmetric Nash. In the cooperative case, *ceteris paribus*, donations are higher, and so is fertility, with respect to the cooperative equilibrium; further, because per capita donations decrease less rapidly in the cooperative than in the noncooperative, the (negative) elasticity of endogenous fertility to an increase in the survival probability of children is smaller (in absolute value) in the cooperative case; more details about the two cases and their properties can be found in Boldrin and Jones (2002).

With respect to (c), we look at two basic case: child care services are produced directly at home (which we label the one sector case) or are produced by the market (the two sector case). In both cases the technology for rearing children is the same, and it uses both goods and time as specified in the previous subsection. The difference between the two models is subtle and has to do with the incentives of parents to invest in the human capital of the children: in the one sector case, by increasing the human capital of their own children, parents affect both their future labor income and their opportunity cost of having children, in so far as the latter requires parental time. In the two sector model, parental investment in the human capital of their own children only increases the labor income of the latter, as the cost of rearing children is determined by market prices, and cannot be influenced by individual variations in parental human capital.

With respect to (b), we consider both the general CES production function  $y = F(k, \tilde{h}) = A [\eta k^\rho + h^\rho]^{1/\rho}$ ,  $-\infty < \rho < 1$ , and the special Cobb-Douglas case  $y = F(k, \tilde{h}) = Ak^\alpha h^{1-\alpha}$   $0 < \alpha < 1$ . The reason for studying also a general CES is that, for the latter, when  $\rho \neq 0$ , variations in factor intensities induce endogenous variations in factor shares. The historical evidence we have examined suggests that sizeable variations in the share of income going to capital, labor, and human capital, did occur in the period under consideration.

Finally, (a) is an obvious requirement for two reasons. First, one always looks for the technically simplest model that can do the job; meaning, in this case, the model with the minimum number of periods that allow facing the data. Our current guess is that models in which individuals live for four periods, of twenty years each, should be enough for our purposes. Second, as our contention is that changes in age specific survival probabilities have substantially different effects on investment, human capital accumulation and fertility, one needs to work with a model that is rich enough to treat variations in age specific mortality rates independently. Furthermore, the historical evidence itself says that the growth in life expectancy was not the outcome of a homogeneous drop in mortality rates at all ages, but that, instead, different mortality rates changed at different points in time.

## 5. THE B&B MODEL WITH ENDOGENOUS GROWTH

**5.1. The Simplest B&B Model.** The Household Problem is given by:

$$U = \sum_{t=0}^{\infty} \beta^t g(N_t^s) u(c_t^s) = \sum_{t=0}^{\infty} \beta^t g(\pi_t N_t^b) u(c_t^s) = \sum_{t=0}^{\infty} \beta^t g(\pi_t N_t^b) u\left(\frac{C_t}{\pi_t N_t^b}\right)$$

subject to:

$$C_t + a_t N_{t+1}^b + \hat{w}_t \hat{\ell}_t^{cc} N_{t+1}^b + H_{t+1} + K_{t+1} \leq w_t N_t^s h_t^s \ell_t^m + \hat{w}_t N_t^s \ell_t^{cc} + R_t K_t^{10}$$

$$\hat{\ell}_t^{cc} \geq b_t$$

$$\ell_t^m + \ell_t^{cc} \leq 1$$

where  $C_t$  is aggregate spending in period  $t$  by the dynasty on consumption,  $N_{t+1}^b$  is the number of births in period  $t$ ,  $H_{t+1}$ , and  $K_{t+1}$  are aggregate spending on the dynasty on human and physical capital in period  $t$ ,  $N_t^s = \pi_t N_t^b$  is the number of surviving members in the dynasty in period  $t$ ,  $\pi_t$  is the effective labor supply per surviving period  $t$  middle aged person,  $h_t^s$  is the human capital per surviving middle aged person

<sup>10</sup>Here we have used the simplifying assumption that  $H$  is produced using the market technology, as discussed below. This amounts to an aggregation assumption— if  $H$  is produced in a separate sector with the same, CRS, production function, this will follow automatically. Many models of human capital investment do not satisfy this assumption, however— often assuming that labor's share is higher in the  $H$  sector. See Lucas (1988) for example. It is of interest to explore other alternatives as well.

in period  $t$ ,  $\hat{\ell}_t^{cc}$  is the number of hours of child care purchased per new child,  $a_t$  is the goods requirement per child by the dynasty in period  $t$ ,  $\ell_t^m$  is the number of hours the typical dynasty member spends in the market sector working,  $\ell_t^{cc}$  is the number of hours the typical dynasty member spends in the childcare sector working,  $R_t$  is the gross return on capital,  $w_t$  is the wage rate for one unit of time with one unit of human capital (so  $w_t h_t^s$  is the wage rate per unit of time for labor with  $h_t^s$  units of human capital), and  $\hat{w}_t$  is the wage rate in the childcare sector.

For  $h_t^s$ , we assume that investments are made for all children surviving to age 5, thus investments per person are  $h_t^s = H_t/\pi_{3t}N_t^b$ . Thus,

$$N_t^s h_t^s = \pi_t N_t^b H_t / \pi_{3t} N_t^b = \frac{\pi_t}{\pi_{3t}} H_t.$$

Firms face static problems:

$$\begin{aligned} \max \quad & Y_t - R_t K_t^f - w_t L_t^f \\ \text{s.t.} \quad & Y_t \leq F(K_t^f, L_t^f) \end{aligned}$$

Goods and Labor market clearing (per dynasty) require that:

$$(F1) \quad K_t^f = K_t$$

$$(F2) \quad L_t^f = N_t^s h_t^s \ell_t^m$$

$$(F3) \quad \hat{\ell}_t^{cc} N_{t+1}^b = b_t N_t^s \ell_t^{cc}$$

$$(F4) \quad C_t + a_t N_{t+1}^b + H_{t+1} + K_{t+1} = Y_t$$

Note that interiority implies immediately that  $\hat{w}_t = w_t h_t^s$  that is, wages are equalized across the two sectors. Because of this, letting  $\theta_t = a_t + b_t \hat{w}_t$ , we can rewrite constraints of the individual household problem as:

$$C_t + a_t N_{t+1}^b + \hat{w}_t N_{t+1}^b \hat{\ell}_t^{cc} + H_{t+1} + K_{t+1} \leq w_t N_t^s h_t^s \ell_t^m + \hat{w}_t N_t^s \ell_t^{cc} + R_t K_t$$

$$C_t + a_t N_{t+1}^b + \hat{w}_t N_{t+1}^b b_t + H_{t+1} + K_{t+1} \leq w_t N_t^s h_t^s \ell_t^m + \hat{w}_t N_t^s \ell_t^{cc} + R_t K_t$$

$$C_t + \theta_t N_{t+1}^b + H_{t+1} + K_{t+1} \leq w_t N_t^s h_t^s \ell_t^m + w_t h_t^s N_t^s \ell_t^{cc} + R_t K_t$$

$$C_t + \theta_t N_{t+1}^b + H_{t+1} + K_{t+1} \leq w_t N_t^s h_t^s \ell_t^m + w_t h_t^s N_t^s (1 - \ell_t^m) + R_t K_t$$

$$C_t + \theta_t N_{t+1}^b + H_{t+1} + K_{t+1} \leq w_t N_t^s h_t^s + R_t K_t$$

$$C_t + \theta_t N_{t+1}^b + H_{t+1} + K_{t+1} \leq w_t \frac{\pi_t}{\pi_{3t}} H_t + R_t K_t.$$

This is the version of the BC we will use below. Thus, the Household Problem is given by:

$$U = \sum_{t=0}^{\infty} \beta^t g(\pi_t N_t^b) u\left(\frac{C_t}{\pi_t N_t^b}\right)$$

subject to:

$$C_t + \theta_t N_{t+1}^b + H_{t+1} + K_{t+1} \leq w_t \frac{\pi_t}{\pi_{3t}} H_t + R_t K_t$$

Note that there is also a conceptual difficulty in assuming that  $a_t$  is positive in an endogenous growth model. If we assume that  $a_t$  is 'constant,' i.e., converges to a constant, de-trended  $a_t \rightarrow 0$ . If we assume that it is growing at some exogenous rate,  $a_t = (1+g)^t a_0$  then if the output of the economy (per capita) grows at a lower rate, then  $a_t N_{t+1}^b / Y_t \rightarrow \infty$  which is not possible. If on the other hand the output of the economy (per capita) grows at a higher rate, then  $a_t N_{t+1}^b / Y_t \rightarrow 0$ , again not very sensible. This is something that comes out of using an endogenous growth model rather than an exogenous one (as in the original B&B model), since there, we can assume that  $(1+g)$  is the exogenously given rate of growth of technology, and neither of these issues arises. The only real solutions to this problem are to either assume that  $a_t = 0$  or to assume that the goods consumption of children enter the utility of the planner. The second option is probably better, since this would guarantee that  $a_t N_{t+1}^b / Y_t$  is a constant fraction of output per person on a BGP (with the right choice of utility function). Since this would raise a host of issues that we do not want to deal with here, we will take the less appealing route and assume that  $a_t = 0$ . Thus,  $\theta_t = b_t \hat{w}_t$ .

Because of this, it follows that feasibility becomes:

$$(F4) \quad C_t + H_{t+1} + K_{t+1} = Y_t.$$

Note that  $\theta_t N_{t+1}^b$  does not appear in this equation! That is,  $\theta_t N_{t+1}^b = b_t w_t h_t^s N_{t+1}^b$  can be thought of as entering the individual dynasty budget constraint since the household could either buy this service, or provide it at home, but, it does not require market output.

Assuming that  $u(c) = c^{1-\sigma}/(1-\sigma)$ , and  $g(N) = N^\eta$ , we can write this problem as:

$$U = \sum_{t=0}^{\infty} \frac{\beta^t}{(1-\sigma)} [\pi_t N_t^b]^\eta \left[ \frac{C_t}{\pi_t N_t^b} \right]^{1-\sigma} = \sum_{t=0}^{\infty} \frac{\beta^t}{(1-\sigma)} [\pi_t N_t^b]^{\eta+\sigma-1} [C_t]^{1-\sigma}$$

subject to:

$$(\beta^t \lambda_t) \quad C_t + N_{t+1}^b \theta_t + H_{t+1} + K_{t+1} \leq w_t \frac{\pi_t}{\pi_{3t}} H_t + R_t K_t$$



First Order Conditions (FOCs, from now on) are:

$$C_t : \quad \beta^t(1 - \sigma)U_t/C_t = \beta^t\lambda_t$$

$$H_t : \quad \beta^t\lambda_t w_t \frac{\pi_t}{\pi_{3t}} = \beta^{t-1}\lambda_{t-1}$$

$$K_t : \quad \beta^t\lambda_t R_t = \beta^{t-1}\lambda_{t-1}$$

$$N_t^b : \quad \beta^t(\eta + \sigma - 1)U_t/N_t^b = \theta_{t-1}\beta^{t-1}\lambda_{t-1}$$

Where,  $U_t = \frac{1}{(1-\sigma)} [\pi_t N_t^b]^{\eta+\sigma-1} [C_t]^{1-\sigma}$

$$C_t : \quad (1 - \sigma)U_t/C_t = \lambda_t$$

$$H_t : \quad w_t \frac{\pi_t}{\pi_{3t}} = R_t$$

$$K_t : \quad \beta R_t \frac{U_t}{U_{t-1}} = \frac{C_t}{C_{t-1}}$$

$$N_t^b : \quad \beta^t(\eta + \sigma - 1)U_t/N_t^b = \theta_{t-1}\beta^{t-1}(1 - \sigma)U_{t-1}/C_{t-1}$$

$$N_t^b : \quad \beta(\eta + \sigma - 1)\frac{U_t}{U_{t-1}} = \theta_{t-1}(1 - \sigma)\frac{N_t^b}{C_{t-1}}$$

$$N_t^b : \quad \beta(\eta + \sigma - 1)\frac{U_t}{U_{t-1}} = \theta_{t-1}(1 - \sigma)\frac{N_{t-1}^b}{C_{t-1}}\frac{N_t^b}{N_{t-1}^b}$$

Now,

$$\frac{U_t}{U_{t-1}} = \frac{\frac{1}{(1-\sigma)}[\pi_t N_t^b]^{\eta+\sigma-1}[C_t]^{1-\sigma}}{\frac{1}{(1-\sigma)}[\pi_{t-1} N_{t-1}^b]^{\eta+\sigma-1}[C_{t-1}]^{1-\sigma}} = \left[\frac{\pi_t}{\pi_{t-1}}\right]^{\eta+\sigma-1} \gamma_N^{\eta+\sigma-1} \gamma_C^{1-\sigma},$$

Thus, assuming that Balanced Growth Path (BGP, from now on) holds, the FOCs become:

$$C_t : \quad (1 - \sigma)U_t/C_t = \lambda_t$$

$$H_t : \quad w_t \frac{\pi_t}{\pi_{3t}} = R_t$$

$$K_t : \quad \beta R \gamma_N^{\eta+\sigma-1} \gamma_C^{1-\sigma} = \gamma_C$$

$$N_t^b : \quad \beta(\eta + \sigma - 1)\gamma_N^{\eta+\sigma-1}\gamma_C^{1-\sigma} = \theta_{t-1}(1 - \sigma)\frac{N_{t-1}^b}{C_{t-1}}\gamma_N$$

So,

$$K_t : \quad \gamma_N^{\eta+\sigma-1}\gamma_C^{1-\sigma} = \frac{\gamma_C}{\beta R}$$

$$N_t^b : \quad \beta(\eta + \sigma - 1)\gamma_N^{\eta+\sigma-1}\gamma_C^{1-\sigma} = \theta_{t-1}(1 - \sigma)\frac{N_{t-1}^b}{C_{t-1}}\gamma_N$$

$$H_t : \quad w_t \frac{\pi_t}{\pi_{3t}} = R_t$$

These are the 3 equations referred to in the title,<sup>11</sup> they come out of individual dynasty optimization plus intertemporal competitive equilibrium. As we will see, increases in  $\frac{\pi_t}{\pi_{3t}}$ , can, at least in some circumstances, generate and increase in the growth rate of per capita output. What are  $R$ ,  $w$  and  $\theta$ ? As discussed above,

$$\theta_t = b_t w_t h_t^s = \frac{b_t w_t H_t}{\pi_{3t} N_t^b}.$$

Note that this implies that either  $\frac{H}{N}$  is constant, or that  $\theta_t$  is not (assuming that  $w_t$  is, along a BGP) even when the parameters  $b_t$  and  $\pi_{3t}$  are constant. Substituting this into the equations above gives:

$$K_t : \quad \gamma_N^{\eta+\sigma-1}\gamma_C^{1-\sigma} = \frac{\gamma_C}{\beta R}$$

$$N_t^b : \quad \beta(\eta + \sigma - 1)\gamma_N^{\eta+\sigma-1}\gamma_C^{1-\sigma} = \frac{b_{t-1}w_{t-1}}{\pi_{3t-1}}\frac{H_{t-1}}{N_{t-1}^b}(1 - \sigma)\frac{N_{t-1}^b}{C_{t-1}}\gamma_N$$

$$\text{or } N_t^b : \quad \beta(\eta + \sigma - 1)\gamma_N^{\eta+\sigma-1}\gamma_C^{1-\sigma} = \frac{b_{t-1}w_{t-1}}{\pi_{3t-1}}\frac{H_{t-1}}{C_{t-1}}(1 - \sigma)\gamma_N$$

$$\text{or } N_t^b : \quad \beta(\eta + \sigma - 1)\gamma_n^{\eta+\sigma-1}\gamma_c^{1-\sigma} = \frac{bw}{\pi_3}\frac{H}{C}(1 - \sigma)\gamma_n$$

$$H_t : \quad w_t \frac{\pi_t}{\pi_{3t}} = R_t$$

Total capital (per dynasty) in period  $t$  is just  $K_t$ . Total effective labor supply (per dynasty) is  $N_t^s h_t^s$  while the amount used for child rearing is  $b_t N_{t+1}^b$  hours in the dynasty. Thus, labor market clearing requires that effective labor supply in the market activity is:

$$\begin{aligned} L_t^m &= h_t^s [N_t^s h_t^s - b_t N_{t+1}^b] = h_t^s [\pi_t N_t^b - b_t N_{t+1}^b] = h_t^s N_t^b \left[ \pi_t - b_t \frac{N_{t+1}^b}{N_t^b} \right] \\ &= \frac{H_t N_t^b}{\pi_{3t} N_t^b} \left[ \pi_t - b_t \frac{N_{t+1}^b}{N_t^b} \right] = H_t \left[ \frac{\pi_t}{\pi_{3t}} - \frac{b_t}{\pi_{3t}} \frac{N_{t+1}^b}{N_t^b} \right], \end{aligned}$$

since,  $h_t^s = \frac{H_t}{\pi_{3t} N_t^b}$ . Note that the term  $\pi_t/\pi_{3t}$  is the conditional

probability of surviving to working age conditional on surviving to age 5; according to our interpretation this is the relevant measure of longevity we take as the main ‘‘cause’’ of the industrial revolution, at least in this simple version of the model. In a more complete version, we also look at the expected length of the period during which dynasty members are of working age.

Thus,

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<sup>11</sup>Indeed, every other model we consider in this paper takes more than three equations to ‘generate’ an industrial revolution.

$$\begin{aligned} w &= F_\ell \left( 1, \frac{H_t}{K_t} \left[ \frac{\pi_t}{\pi_{3t}} - b_t \gamma_n \right] \right) = F_\ell \left( 1, \frac{H}{K} \left[ \frac{\pi}{\pi_3} - b \gamma_n \right] \right) \\ R &= F_k \left( 1, \frac{H_t}{K_t} \left[ \frac{\pi_t}{\pi_{3t}} - b_t \gamma_n \right] \right) = F_k \left( 1, \frac{H}{K} \left[ \frac{\pi}{\pi_3} - b \gamma_n \right] \right) \end{aligned}$$

Finally, we must also have:

$$C_t + H_{t+1} + K_{t+1} = F \left( K_t, H_t \left[ \frac{\pi_t}{\pi_{3t}} - \frac{b_t}{\pi_{3t}} \frac{N_{t+1}^b}{N_t^b} \right] \right),$$

Dividing by  $H_t$  gives:

$$\frac{C_t}{H_t} + \frac{H_{t+1}}{H_t} + \frac{K_{t+1}}{H_t} = F \left( \frac{K_t}{H_t}, \left[ \frac{\pi_t}{\pi_{3t}} - \frac{b_t}{\pi_{3t}} \frac{N_{t+1}^b}{N_t^b} \right] \right)$$

or, assuming BGP,

$$\frac{C}{H} + \gamma_H + \frac{K}{H} \gamma_H = F \left( \frac{K}{H}, \left[ \frac{\pi}{\pi_3} - \frac{b}{\pi_3} \gamma_N \right] \right)$$

Thus, the system of equations that we want to solve is:

$$K_t : \quad \gamma_N^{\eta+\sigma-1} \gamma_C^{1-\sigma} = \frac{\gamma_C}{\beta R}$$

$$\text{or,} \quad \gamma_C^\sigma \gamma_N^{1-\eta-\sigma} = \beta R$$

$$\text{or,} \quad \left[ \frac{\gamma_C}{\gamma_N} \right]^\sigma \gamma_N^{1-\eta} = \beta R$$

(see the relevance of  $\left[ \frac{\gamma_C}{\gamma_N} \right]$  below).

$$N_t^b : \quad \beta(\eta + \sigma - 1) \gamma_N^{\eta+\sigma-1} \gamma_C^{1-\sigma} = \frac{bw}{\pi_3} \frac{H}{C} (1 - \sigma) \gamma_N$$

$$H_t : \quad w \frac{\pi}{\pi_3} = R$$

$$w : \quad w = F_\ell \left( 1, \frac{H}{K} \left[ \frac{\pi}{\pi_3} - b \gamma_n \right] \right)$$

$$R : \quad R = F_k \left( 1, \frac{H}{K} \left[ \frac{\pi}{\pi_3} - b \gamma_n \right] \right)$$

$$Feas : \quad \frac{C}{H} + \gamma_H + \frac{K}{H} \gamma_H = F \left( \frac{K}{H}, \left[ \frac{\pi}{\pi_3} - \frac{b}{\pi_3} \gamma_N \right] \right)$$

Variables are:  $R, w, C/H, K/H, \gamma_H, \gamma_N, \gamma_C$ . Note that  $\gamma_C = \gamma_H$  (or  $C/H$  and/or  $K/H$  must go to zero) since  $\gamma_C$  is the growth rate of  $C$  not  $c$ . These are the growth rates in aggregates. Thus, the growth rates in per capita terms are given by:

$$\gamma_c = \frac{c_{t+1}^s}{c_t^s} = \frac{C_{t+1}/N_{t+1}^s}{C_t/N_t^s} = \frac{C_{t+1}}{C_t} \frac{N_t^s}{N_{t+1}^s} = \gamma_C/\gamma_N$$

Again, what we want to do with this is to do comparative statics with respect to changing  $\pi$ , and remember that  $b$  depends on the whole array of  $\pi$ 's.

**5.2. B&B as a Planning Problem.** Rewriting the B&B Competitive Equilibrium Problem as a Planner's Problem, we obtain (assuming that  $u(c) = c^{1-\sigma}/(1-\sigma)$ , and  $g(N) = N^\eta$ ):

$$U = \sum_{t=0}^{\infty} \frac{\beta^t}{(1-\sigma)} [\pi_t N_t^b]^\eta \left[ \frac{C_t}{\pi_t N_t^b} \right]^{1-\sigma} = \sum_{t=0}^{\infty} \frac{\beta^t}{(1-\sigma)} [\pi_t N_t^b]^{\eta+\sigma-1} [C_t]^{1-\sigma}$$

subject to:

$$(\beta^t \lambda_t) \quad C_t + H_{t+1} + K_{t+1} \leq F \left[ K_t, (\pi_t N_t^b) h_t \left( 1 - b \frac{N_{t+1}^b}{\pi_t N_t^b} \right) \right]$$

Here, the second term,  $(\pi_t N_t^b) h_t \left( 1 - b \frac{N_{t+1}^b}{\pi_t N_t^b} \right)$  is derived from the fact that there are  $(\pi_t N_t^b)$  number of people surviving to working age, with  $h_t$  human capital per surviving person, and that  $\left( 1 - b \frac{N_{t+1}^b}{\pi_t N_t^b} \right)$  is the amount of time remaining, per survivor, after the time spent doing child care for the next generation is accounted for.

Human capital per worker is  $h_t = \frac{H_t}{\mu_t N_t^b}$ , this gives us a labor input

$$\begin{aligned} L_t &= (\pi_t N_t^b) h_t \left( 1 - b \frac{N_{t+1}^b}{\pi_t N_t^b} \right) \\ &= (\pi_t N_t^b) \frac{H_t}{\mu_t N_t^b} \left( \frac{\pi_t N_t^b - b N_{t+1}^b}{\pi_t N_t^b} \right) \\ &= \frac{\pi_t}{\mu_t} H_t \left( \frac{\pi_t N_t^b - b N_{t+1}^b}{\pi_t N_t^b} \right) \end{aligned}$$

Thus, the constraint for the problem is:

$$(\beta^t \lambda_t) \quad C_t + H_{t+1} + K_{t+1} \leq F \left[ K_t, \frac{\pi_t}{\mu_t} H_t \left( \frac{\pi_t N_t^b - b N_{t+1}^b}{\pi_t N_t^b} \right) \right]$$

or

$$(\beta^t \lambda_t) \quad C_t + H_{t+1} + K_{t+1} \leq F [K_t, L_{1t} h_t]$$

and

$$bN_{t+1}^b \leq L_{2t}$$

$$L_{1t} + L_{2t} \leq \pi_t N_t^b$$

Which can be written as:

$$(\beta^t \lambda_t) \quad C_t + H_{t+1} + K_{t+1} \leq F \left[ K_t, L_{1t} \frac{H_t}{\mu_t N_t^b} \right]$$

and

$$bN_{t+1}^b \leq L_{2t}$$

$$L_{1t} + L_{2t} \leq \pi_t N_t^b$$

TO BE ADDED: SHOW THAT PLANNER PROBLEM AND EQUILIBRIUM PROBLEM HAVE EQUIVALENT SOLUTIONS.

## 6. THE B&J MODEL WITH ENDOGENOUS GROWTH

**6.1. Assumptions and Notation.** The model studied in this part has T-period lives (T will be either 3 or 4, for the time being), altruism from children toward parents but not viceversa, and certainty-equivalence within each family (i.e., we look at a "representative family" and make the bold assumption that both an economy-wide insurance for individual death risk, and bequest mechanisms of the specific form specified below, are automatically implemented.) Utility is arbitrary, in principle, but we specialize it to CES to get analytical solutions and then calibrate the model. Population in each period is denoted, respectively,  $N^y$  for the young,  $N^m$  for middle age ( $N^{m1}$  and  $N^{m2}$  in the model with four periods, distinguishing between "early" and "late" middle age), and  $N^o$  for the old. Age specific survival probabilities to the next period of life are denoted as  $\pi^y$ ,  $\pi^m$ , and  $\pi^o$  for young, early middle-age and late middle-age people, respectively; they range in the interval  $[0, 1]$ , with  $\pi^o = 0$  in the three period model.

Production function is arbitrary neoclassical, but we assume either Cobb-Douglas or general CES to make progresses with algebra. As argued in the motivational section, the empirically most relevant case appears to be that of a CES with an elasticity of substitution between human and physical capital less than one, i.e., with less substitutability than the Cobb-Douglas. The aggregate production function is denoted as  $Y_t = F(K_t, H_t) = L_t F(k_t, h_t)$ , where  $L_t$  is the total amount of raw labor supplied in period  $t$ , and  $F$  has the usual neoclassical properties. People can only accumulate human capital during the first period, when young. Two different technologies to accumulate human capital will be

considered. In the first, labelled the *one sector case*, for each child parents invest goods, in a quantity  $a \geq 0$ , and a fraction  $b \in [0, 1]$  of their *own time* to rear her to working age. In the second, the *two sector case*, both goods and time are purchased on the market, from a competitive child-care sector that uses this same technology.

People can procreate only during their middle age (early middle age, when  $T=4$ ), and the number of newborns in period  $t$ , per middle age person, is  $n_t^b$ . The total cost of having a child is denoted  $\theta_t + \mu_t h_{t+1}$ , where  $\theta_t = a + b_t w_t h_t$ ,  $\mu_t \in [\pi_t^y, 1]$  is the exogenous probability that a child born in period  $t$  survives till when she can receive professional training/education, and  $h_{t+1}$  is her endogenous level of human capital, chosen by the parents.

Notice that, with this timing of events and, in particular, under the assumption that old people do not work, the role played by the survival probability  $\pi^m$  in determining the investment in human capital is ambiguous. For sure, when  $\pi^m$  increases, total investment by middle age agents should increase. Total investment, though, can take three forms: more children, more capital per worker, and a higher human capital for each child. Of these three kinds of investment, the first decreases the (growth rate of) per capita income, while the other two should *both* increase it. The conjecture is, therefore, that an increase in  $\pi^m$  will lead to higher aggregate investment in all three directions ( $k$ ,  $n$  and  $h$ ) with the increase in  $k$  and in the  $h/k$  ratio depending on relative values of the technological parameters. An increase in  $\pi^y$ , which, as we know from previous work, leads to less fertility, should increase  $h$  unambiguously, while it is unclear if it leads also to higher levels of  $k$ ; intuition and basic algebra suggests that, in any case, the net effect on the  $h/k$  ratio should be positive.

**6.2. The Simplest B&J Model.** Here we illustrate the analytically simplest version of the B&J model, which is the one adopted so far in the calibration exercises summarized in the next section. This version has three period lives, positive mortality only for the young, middle age agents behaving non-cooperatively in determining donations to parents, a Cobb-Douglas production function, and two distinct sectors, one for market output and for childcare services. In this case, parents do not use their own time to rear children but purchase the services from the market, at the going equilibrium price. The optimization problem is

$$\text{Max}_{\{n_t^b, d_t^i, h_{t+1} k_{t+1}\}} u(c_t^m) + \zeta u(c_t^o) + \beta u(c_{t+1}^o)$$

subject to:

$$c_t^m + k_{t+1} + n_t^b(a_t + \hat{w}_t \hat{\ell}_t^{cc} + \mu_t h_{t+1}) + d_t^i \leq w_t h_t \ell_t^m + \hat{w}_t \ell_t^{cc}$$

$$\begin{aligned}
 c_t^o &\leq R_t k_t + d_t^i + D_t \\
 c_{t+1}^o &\leq R_{t+1} k_{t+1} + \pi_t n_t^b d_{t+1} \\
 \ell_t^m + \ell_t^{cc} &\leq 1 \\
 \hat{\ell}_t^{cc} &\geq b_t.
 \end{aligned}$$

Market clearing in the childcare sector implies that  $\hat{\ell}_t^{cc} = \ell_t^{cc}$ , for the representative agent in the cohort. Similarly, interiority will imply that  $\hat{w}_t = h_t w_t$ , and hence, it follows that  $\theta_t = a_t + b_t h_t w_t$ , and thus, since  $a_t = 0$ ,  $\theta_t = b_t h_t w_t$ . Thus, output, in period  $t + 1$ , per old person in the same period, is given by:

$$y_{t+1} = F(k_{t+1}, \tilde{h}_{t+1}) = F[k_{t+1}, \pi_t n_t^b h_{t+1} (1 - b_t n_{t+1}^b)],$$

and rental rates and wage rates are given by:

$$R_{t+1} = F_k(k_{t+1}, \pi_t n_t^b h_{t+1} (1 - b_t n_{t+1}^b))$$

and,

$$w_{t+1} = F_h(k_{t+1}, \pi_t n_t^b h_{t+1} (1 - b_t n_{t+1}^b))$$

respectively.

6.2.1. *Donations and Old Age Consumption.* Assuming that  $u(c) = c^{1-\sigma}/(1-\sigma)$ , first order conditions for donations are:

$$c_t^o = \zeta^{1/\sigma} c_t^m,$$

from which we find that:

$$\begin{aligned}
 d_t &= \frac{\zeta^{1/\sigma} (w_t h_t - I_t) - R_t k_t}{\zeta^{1/\sigma} + \pi_{t-1} n_{t-1}^b} \\
 c_t^o &= \frac{\zeta^{1/\sigma}}{\zeta^{1/\sigma} + \pi_{t-1} n_{t-1}^b} [\pi_{t-1} n_{t-1}^b (w_t h_t - I_t) + R_t k_t]. \\
 c_t^m &= \frac{1}{\zeta^{1/\sigma} + \pi_{t-1} n_{t-1}^b} [\pi_{t-1} n_{t-1}^b (w_t h_t - I_t) + R_t k_t] \\
 \frac{\partial c_t^o}{\partial s_{t-1}} &= \frac{\zeta^{1/\sigma}}{\zeta^{1/\sigma} + \pi_{t-1} n_{t-1}^b} R_t \\
 \frac{\partial c_t^o}{\partial n_{t-1}^b} &= \frac{\pi_{t-1} \zeta^{1/\sigma}}{[\zeta^{1/\sigma} + \pi_{t-1} n_{t-1}^b]^2} [\zeta^{1/\sigma} (w_t h_t - I_t) - R_t k_t] \\
 \frac{\partial c_t^o}{\partial h_t} &= \frac{\zeta^{1/\sigma}}{\zeta^{1/\sigma} + \pi_{t-1} n_{t-1}^b} \pi_{t-1} n_{t-1}^b w_t
 \end{aligned}$$

6.2.2. *Intertemporal First Order Conditions.* (FOCn)

$$u'(c_t^m)(\theta_t + \mu_t h_{t+1}) = \beta u'(c_{t+1}^o) \frac{\partial c_{t+1}^o}{\partial n_t^b}$$

(FOCh)

$$u'(c_t^m) \mu_t n_t^b = \beta u'(c_{t+1}^o) \frac{\partial c_{t+1}^o}{\partial h_{t+1}}$$

(FOCk)

$$u'(c_t^m) = \beta u'(c_{t+1}^o) \frac{\partial c_{t+1}^o}{\partial s_t}$$

Reducing the system to two equations and two unknowns. From (FOCh) and (FOCk) we observe that,

$$\mu_{t-1} F_k(k_t, \tilde{h}_t) = \pi_{t-1} F_h(k_t, \tilde{h}_t).$$

Using  $y = Ak^\alpha \tilde{h}^{1-\alpha}$ , the latter equality yields

$$n_t^b = \frac{1}{b_t} \left( 1 - \frac{1-\alpha}{\alpha \mu_{t-1} n_{t-1}^b} \hat{k}_t \right),$$

So, because  $n_{t-1}^b$  is predetermined, the intertemporal equilibrium relation between capital intensity and fertility is one to one. We also have

$$\begin{aligned} R_t &= \alpha A \left[ \frac{\pi_{t-1}(1-\alpha)}{\alpha \mu_{t-1}} \right]^{1-\alpha} \\ w_t &= (1-\alpha) A \left[ \frac{\alpha \mu_{t-1}}{\pi_{t-1}(1-\alpha)} \right]^\alpha \\ \hat{y}_t &= A \left( \frac{\pi_{t-1}(1-\alpha)}{\alpha \mu_{t-1}} \right)^{1-\alpha} \hat{k}_t = \hat{A}_t \hat{k}_t \\ \frac{\hat{h}_t}{h_t} &= \frac{\tilde{h}_t}{h_t} = \frac{(1-\alpha)\pi_{t-1}}{\alpha \mu_{t-1}} \hat{k}_t. \end{aligned}$$

$$\hat{I}_t = \hat{k}_{t+1} + \frac{1}{b} \left( 1 - \frac{1-\alpha}{\alpha \mu_{t-1} n_{t-1}^b} \hat{k}_t \right) (wb + \mu_t \gamma_{h,t})$$

The remaining variables can be computed accordingly.



6.2.3. *Computing the Intertemporal Competitive Equilibrium.* We exploit the consumption ratio

$$\frac{\widehat{c}_t^o}{\widehat{c}_t^m} = \zeta^{1/\sigma}.$$

When the latter is plugged in FOCk, this can be solved to express the growth rate of per capita output as a function of parameters and state variables only:

$$\gamma_{h,t}^\sigma = \frac{\beta \zeta^{1/\sigma-1}}{\zeta^{1/\sigma} + \frac{\pi_t}{b} \left(1 - \frac{1-\alpha}{\alpha \mu_{t-1} n_{t-1}^b} \widehat{k}_t\right)} \alpha A \left[ \frac{\pi_t (1-\alpha)}{\alpha \mu} \right]^{1-\alpha}$$

From the latter, the whole competitive equilibrium sequence can be computed recursively starting from given initial conditions for  $\{k_0, h_0, n_{-1}\}$  and parameters.

6.2.4. *BGP Solutions.* Because,

$$\begin{aligned} \widehat{c}^o &= \frac{\zeta^{1/\sigma}}{\zeta^{1/\sigma} + \pi n} \left[ \pi n (w - \widehat{I}) + R \widehat{k} \right]. \\ \widehat{c}^m &= \frac{1}{\zeta^{1/\sigma} + \pi n} \left[ \pi n (w - \widehat{I}) + R \widehat{k} \right] \end{aligned}$$

we have that.

$$\frac{\widehat{c}^o}{\widehat{c}^m} = \zeta^{1/\sigma}$$

The eight variables to be determined are  $\{\widehat{d}, \widehat{k}, n, \gamma_h, \widehat{c}^m, \widehat{c}^o, R, w\}$ . Starting with the donation function, the eight BGP equations we use are

$$d = \frac{\zeta^{1/\sigma} (w - \widehat{I}) - R \widehat{k}}{\zeta^{1/\sigma} + \pi n}$$

FOCn

$$[\gamma_h]^{\sigma-1} (bw + \mu \gamma_h) = \beta \frac{\pi \zeta^{1/\sigma-1}}{[\zeta^{1/\sigma} + \pi n]^2} \left[ \zeta^{1/\sigma} (w - \widehat{I}) - R \widehat{k} \right]$$

FOCh

$$[\gamma_h]^\sigma \mu = \beta \frac{\pi \zeta^{1/\sigma-1}}{\zeta^{1/\sigma} + \pi n} w$$

FOCk

$$[\gamma_h]^\sigma = \beta \frac{\zeta^{1/\sigma-1}}{\zeta^{1/\sigma} + \pi n} R$$

BCMa

$$\widehat{c}^m + \gamma_h \widehat{k} + n(bw + \mu \gamma_h) + \widehat{d} = w$$

BCold

$$\widehat{c}^o = R\widehat{k} + \pi n\widehat{d}$$

MCK

$$R = F_k(\widehat{k}, \pi n(1 - bn))$$

MCH

$$w = F_h(\widehat{k}, \pi n(1 - bn))$$

These can be manipulated, as in the intertemporal competitive equilibrium case, to find that

$$\begin{aligned}\widehat{k} &= \frac{\alpha\mu n(1 - bn)}{1 - \alpha} \\ R &= \alpha A \left[ \frac{\pi(1 - \alpha)}{\alpha\mu} \right]^{1-\alpha} \\ w &= (1 - \alpha)A \left[ \frac{\alpha\mu}{\pi(1 - \alpha)} \right]^\alpha \\ \widehat{y} &= A \left( \frac{\pi(1 - \alpha)}{\alpha\mu} \right)^{1-\alpha} \widehat{k} = \widehat{A}k_t\end{aligned}$$

Last step, using FOC<sub>k</sub> solve for  $\gamma_h$  as a function of  $n$  and then use FOC<sub>n</sub> to solve the nonlinear equation for the BGP value of  $n$ .

$$[\gamma_h]^\sigma = \frac{\beta\zeta^{1/\sigma-1}}{\zeta^{1/\sigma} + \pi n} \alpha A \left[ \frac{\pi(1 - \alpha)}{\alpha\mu} \right]^{1-\alpha}$$

## 7. QUANTITATIVE IMPLEMENTATIONS OF THE MODELS

In this section, we describe the results of calibrating and simulating some (two, at the time being) among the simple models described in the preceding sections. To this point in time, we have only conducted full blown simulations of the Barro and Becker version of the model, and “comparative BGP simulations” of one version of the B&J model, the one with two sector, Cobb-Douglas production function and non-cooperative behavior of siblings in the donations game.

What we find in the case of the B&B framework, is that versions of the model that are calibrated to realistic values of the parameters give rise to quantitatively interesting and realistic changes in the rate of growth of output per capita when survival probabilities are changed as they have changed historically in the UK between 1700 and 2000. This is, we think, evidence that the mechanism we are highlighting here – improved survival rates increasing the rate of return to human capital investments and increasing the technological frontier in such a way as to increase the growth rate of output – is worthy of more careful

development. The model does not do so well when it comes to fertility decisions per se, however. Here, in contrast to what is seen in both the time series data for the UK and in the country cross section data described in Section 2, increased survival rates permanently increase fertility rates and thereby asymptotic population growth rates. Although not huge, these increases are of significant size economically. In particular, what the various versions of the models we have simulated so far miss is the following historical demographic pattern: fertility does neither increase nor decrease to a large extent when survival rates (among adults, especially) begins to improve; which leads to a first long period of increasing growth rates of population. Eventually, and especially when children and infant mortality rates drop, fertility also drops rapidly and dramatically, slowing down population growth. Should we extend the analysis to the whole of the twentieth century, then one should require the model to yield a zero rate of population growth. The demographic dynamics generated by the B&B model does not satisfy all such requirements, as population growth rates are always increasing together with the growth rate of per capita GDP as mortality rates decrease. As we show below, when calibrated to (second half of the) XX-century-like parameter values, the B&B model yields a BGP with an endogenous growth rate of output per capita equal to about 2%, which is pretty much what we seem to observe, and a zero growth rate of the population, which is also what we seem to observe.

The behavior of the B&J model is, in some sense, the mirror image of the B&B: it does better on the side of fertility, but more poorly on the side of output, as it tends to exaggerate the increase in the the growth rate of output that follows a drop in mortality. We have so far calibrated and simulated only the non-cooperative version, hence things may appear different once we succeed simulating the transition path for the cooperative version. In the same circumstances as the B&B model, i.e. when calibrated to XX-century-like values, it generates a 2% growth rate of output per capita but fails to achieve a constant population, predicting instead a population that declines at the rather rapid pace of 8% per year.

**7.1. Calibrating the B&B Model.** Recall that the B&B version of the model can be summarized through the choice of utility function, the production function for output and the cost of newborn children. In keeping with the above, we choose:

$$U = \sum_{t=0}^{\infty} \frac{\beta^t}{(1-\sigma)} [\pi_t N_t^b]^\eta \left[ \frac{C_t}{\pi_t N_t^b} \right]^{1-\sigma} = \sum_{t=0}^{\infty} \frac{\beta^t}{(1-\sigma)} [\pi_t N_t^b]^{\eta+\sigma-1} [C_t]^{1-\sigma}$$

and

$$F(K, L) = AK^\alpha(Lh)^{1-\alpha}$$

Thus, the parameters we need to pick are:  $\beta, \sigma, \eta, b, A, \alpha$ . For this, we choose  $\alpha = 0.36$  following the RBC literature and select combinations of the remaining parameters,  $\beta, \sigma, \eta, b$ , and  $A$  so as to match interest rates, population growth rates and output growth rates in the UK in 1750 given the survival rates that were in effect at that time. Since it is not obvious whether calibrating to early in the period (e.g., 1750) or later (e.g., current) is more reasonable, we constructed two versions of the calibration, one matching the 1750 facts, the other matching the facts as they are currently.

To do this, we must first choose the length of a period. For this, we choose  $T = 20$  years as a baseline. Given this, we need data on the survival rates to age 5 and 20. Unfortunately, as can be seen from the data presented in Figure 1C, the Wrigley, et. al., data can only be used to calculate directly survival probabilities to age 15, not 20, for the period 1750 to 2000. The data from [www.mortality.org](http://www.mortality.org) (from 1840 to 1905) and Mitchell (from 1905 to 1970) can be used to calculate survival rates to 20 directly. Survival rates to age 5 are directly available from these sources and can be used to construct a continuous series for the entire 1750 to 2000 period. The data from [www.mortality.org](http://www.mortality.org) can be used to compute estimated survival probabilities to both age 15 and 20, and these are shown in Figure 1C. To adjust for the fact that data on survival to age 20 is not available for the 1750 to 1840 period, we use the fact that the ratio of the probability of survival to age 20 to the probability of survival to age 15 was .96 in 1840, and rose steadily to 1.0 over the period from 1840 to 1970. Thus, using the linear function

$$\pi_t = 0.96 * Surv15W\&S_t$$

seems a conservative approximation. This gives an initial value of  $\pi_{1750} \approx 0.6$  which is what we use for both our calibration and our quantitative exercises. That  $\mu_{1750} \approx 0.75$  comes directly from the Wrigley, et. al. tables. Summarizing then, the actual values of  $\mu_t$  and  $\pi_t$  that we use are given in Table 6 below.

**Table 6: Survival Probabilities**

| Year | $\mu_t$ | $\pi_t$ |
|------|---------|---------|
| 1735 | 0.696   | 0.609   |
| 1765 | 0.744   | 0.666   |
| 1795 | 0.746   | 0.678   |
| 1825 | 0.771   | 0.705   |
| 1850 | 0.728   | 0.660   |
| 1870 | 0.735   | 0.684   |
| 1890 | 0.770   | 0.733   |
| 1910 | 0.829   | 0.824   |
| 1930 | 0.919   | 0.919   |
| 1950 | 0.962   | 0.961   |
| 1970 | 0.977   | 0.976   |

Over the period from 1650 to 1750, population in the UK grew at about 0.1% per year (Wrigley, et. al. (\*\*\*) while per capita GDP grew at about 0.3% per year (Clark (\*\*\*)). Thus, for our 1700 calibration, we will choose parameters so that on an initial BGP,  $\gamma_y = \gamma_n \approx 1.0$  given that  $\mu_{1750} = 0.75$ ,  $\pi_{1750} = 0.60$  (Wrigley and Schofield (\*\*\*)). We choose  $\beta = 1.03^{-1}$  on an annual basis so that real rates of interest with constant consumption would be about 3%. This gives us 2 extra degrees of freedom that we will remove in future vintages of the paper. We adopt a similar strategy for the 2000 calibration. Again we use  $\beta = 1.03^{-1}$  and use the more recent values of  $\mu_{2000} = \pi_{2000} = 1.0$ , and  $\gamma_n \approx 1.0$ ,  $\gamma_y = 1.02$ . The values of the parameters we have chosen for our base case are shown in Table 7.

**Table 7: Calibration of the B&B Model**

|              | 1750 Calibration | 2000 Calibration |
|--------------|------------------|------------------|
| $\alpha$     | 0.36             | 0.36             |
| $\beta$      | $(1.03)^{-1}$    | $(1.03)^{-1}$    |
| $\eta$       | 0.20             | 0.20             |
| $\sigma$     | 0.85             | 0.85             |
| $b$          | 0.06             | 0.10             |
| $A$          | 4.0              | 4.65             |
| $\mu_{1750}$ | 0.75             | 0.75             |
| $\pi_{1750}$ | 0.60             | 0.60             |
| $\mu_{2000}$ | 1.00             | 1.00             |
| $\pi_{2000}$ | 1.00             | 1.00             |

Table 8 shows the targeted quantities and how close the calibrated version of the model comes to matching them.

**Table 8: Model versus Data, Calibrated Moments**

| Target     | 1750 Calibration | Data 1750 | Source | 2000 Calibration | Data 2000 |
|------------|------------------|-----------|--------|------------------|-----------|
| $R$        | 2.99%            | 4.5%      | C&H    | 4.52%            | 4%        |
| $\gamma_n$ | 1.001            | 1.001     | W&S    | 1.001            | 1.003     |
| $\gamma_y$ | 1.001            | 1.003     | Clark  | 1.018            | 1.02      |

## 7.2. Results From the Calibration.

Figures 3C and 3D show the results of the predicted time series of population and per capital income, both from the model and the actual time series from the UK, and both for the 1750 and the 2000 calibrations. For the model quantities, we show BGP levels only, not a full transition in the probabilities. Not surprisingly, they show several common features:

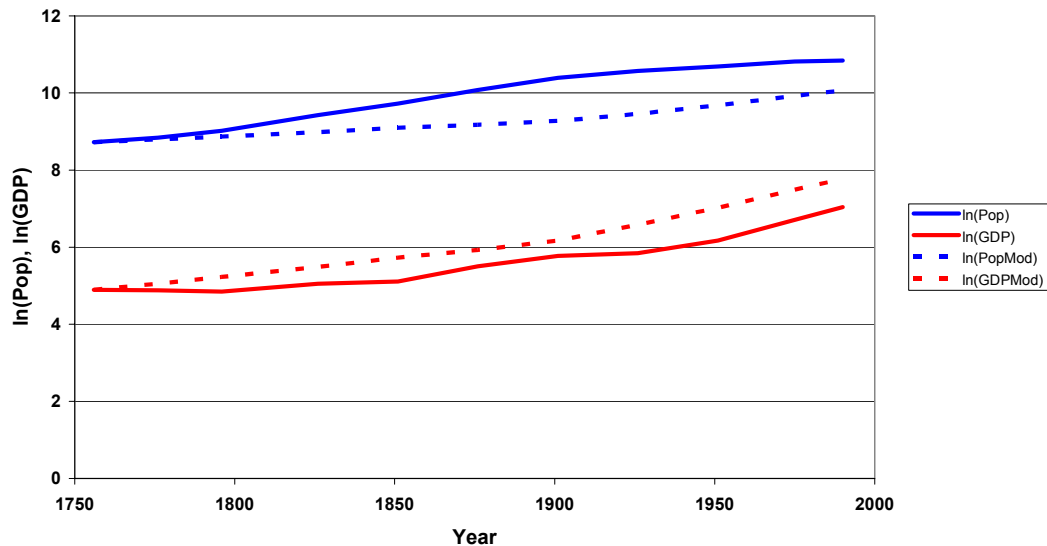
**Figure 3C: Model vs. Data; 1750 Calibration, Levels**

FIGURE 7.1

- (1) When calibrated to either the 1750 or the 2000 BGP, the model is able to match the targeted values with reasonable parameter values.
- (2) When simulated ‘forward’ (or ‘backward’, from 2000) the model also tracks reasonably well the observed changes in the growth rate of output at low frequencies. If anything, it predicts more growth in income than observed.

- (3) The model does not do well, indeed it performs rather poorly, in tracking the overall pattern of population growth rates. For both calibrations, the model predicts an increase in  $\gamma_n$  of about 1% per year between 1750 and 2000, whereas in the data,  $\gamma_{n,1750} \approx \gamma_{n,2000} \approx 1.00$ . This gets at the mechanism that is driving the model results and points to both its strengths and its weaknesses. That is, in the model, increases in survival probabilities increase the rate of return to human capital formation, thus increasing output growth rates. For the same reason however, children become better investments. This increases fertility and thereby increases population growth rates permanently.
- (4) The model is weak at tracking the higher frequency movements in output growth and population growth. This is perhaps not surprising since the driving force is only survival probabilities and it is unlikely that reductions in these are the cause of the Great Depression or the two World War.

**7.3. Calibrating the B&J Model.** As with the B&B model, we need to calibrate an aggregate production function and a utility function that has three arguments, own middle and old age consumption, and consumption of parents. Thus, the parameters we need to pick are:  $\beta, \sigma, \varsigma, b, A, \alpha$ . For this, we choose  $\alpha = 0.36$  following the RBC literature and select combinations of the remaining parameters,  $\beta, \sigma, \eta, b$ , and  $A$  so as to match observables for either 1750 or 2000. As the mortality data used are the same as for the B&B, the description is not repeated here. The values of the parameters we have chosen for our base case are shown in Table 9, while Table 10 shows the targeted quantities and how close the model comes to matching them .

**Table 9: Calibration of the B&J Model**

|              | 1750 Calibration | 2000 Calibration |
|--------------|------------------|------------------|
| $\alpha$     | 0.36             | 0.36             |
| $\beta$      | $(1.03)^{-1}$    | $(1.03)^{-1}$    |
| $\varsigma$  | 1.57             | 1.80             |
| $\sigma$     | 0.50             | 0.50             |
| $b$          | 0.06             | 0.06             |
| $A$          | 7.9              | 10.00            |
| $\mu_{1750}$ | 0.75             | 0.75             |
| $\pi_{1750}$ | 0.60             | 0.60             |
| $\mu_{2000}$ | 1.00             | 1.00             |
| $\pi_{2000}$ | 1.00             | 1.00             |

**Table 10: Model versus Data, Calibrated Moments**

| Target     | 1750 Calibration | Data 1750 | Source | 2000 Calibration | Data 2000 |
|------------|------------------|-----------|--------|------------------|-----------|
| $R$        | 6.56%            | 4.5%      | AK     | 8.6%             | 4%        |
| $\gamma_n$ | 1.001            | 1.001     | W&S    | 1.0009           | 1.003     |
| $\gamma_y$ | 1.0006           | 1.003     | Clark  | 1.019            | 1.02      |

**7.4. Results From the Calibration.** The results for the B&J model are, in some sense, similar but complementary to those of the B&B model. While both can be calibrated to match targeted quantities at either 1750 or 2000, both appear to have troubles at matching observed dynamic patterns. The B&J model, at least in the cooperative version adopted here, seems to have an additional troubles, i.e. it requires an unreasonably high value for the parameter  $\varsigma$  to match observed data. Without a weight that large on the utility of parents, the rate of return on having children is too low, compared to that of just investing in physical capital, and parents end up having an unrealistically low number of children.<sup>12</sup>

Moving to the dynamics, for the B&B it is population that behaves poorly, while for the B&J it seems to be output. This can be seen, more clearly, in Figure 4D, where the B&J model substantially overpredicts the growth rate of output. Notice that this may not be a ‘structural’ weakness of the B&J model, but may steam from the calibration of  $A$ , which, as noted in the previous footnote, is very high and yields a rate of return on capital, in both calibrations, that is substantially higher than the value in the data. In other words, for reasons that still escape us, investing in capital is too much profitable respect to investing in children in the B&J model, hence the mismatch with data.

## 8. DIRECTIONS FOR FUTURE WORK

The experiments run so far point to several potentially fruitful further directions for research.

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<sup>12</sup>We admit being puzzled by the behavior of the B&J model along this dimension, which makes us suspicious of our own algebra. This is because we must, at the same time, use a very large value for  $A$  in the production function, which generates an abnormally high rate of return on capital, and hence the problem with children’s donations noted in the text.



Figure 4C: B-J Model vs. Data; 1750 Calibration, Levels

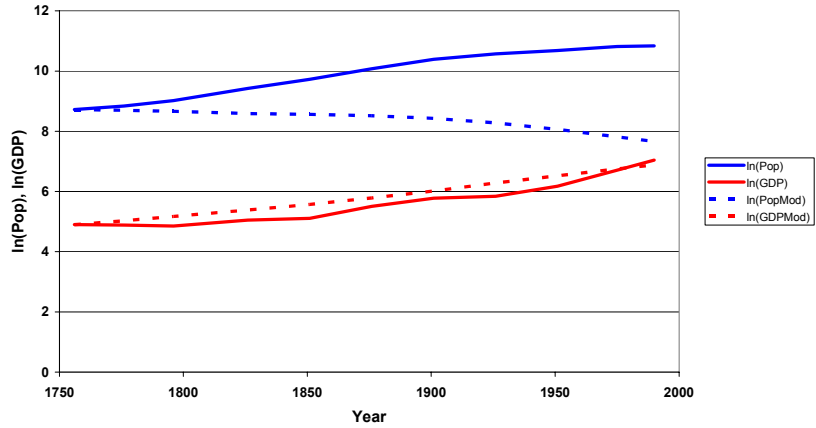


FIGURE 7.2

Figure 4D: B-J Model vs. Data; 2000 Calibration, Levels

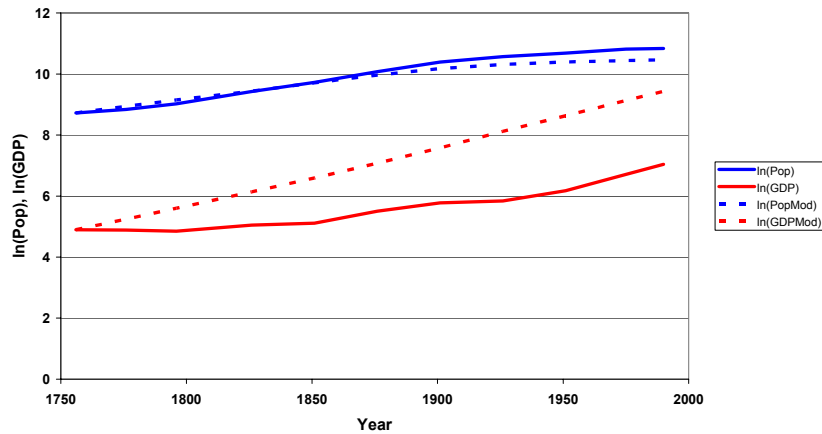


FIGURE 7.3

- (1) Get a version of the B&J model working without assuming an unreasonably large weight for the utility of the old parents. Based on past experience with these models, it is quite possible that this will offer improvements in the tracking of fertility. Aside for possible mathematical mistakes, we expect the cooperative version to make investing in children much more attractive, allowing a more realistic calibration of the production function and, hence, of the rate of return on capital.

- (2) Do the full model with transition dynamics (for both the B&B and B&J models).
- (3) Explore other, more complex models of the formation of human capital and the changes that a CES production function may induce in factor shares and the rate of return on capital.
- (4) Add one more period (two, for the B&B) in the life times of the representative agents, and introduce the relevant mortality rates in the transition from one period to the other. This latter is a particularly important improvement, as it may generate a temporary increase in fertility in the first part of the transition (when the survival rate of adults increases) and a subsequent decrease in fertility later on (when the survival rates of infant and children increase). While this may not eliminate the structural tendency of the B&B model to make output and population growth rates increase together, it may allow the B&J model to better match the historical patterns.

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