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Public education and capital accumulation☆

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Abstract

I study an overlapping generations model where physical and human capitals are inputs of production that can be accumulated by withholding resources from current consumption. Human capital is the output of a schooling system which can be financed either by private expenditures, or by taxes, or by a combination of both. In a political equilibrium with majority voting, public school financing turns out to be an instrument to solve a 'free rider problem'. By improving the skills of next period's workers it increases the expected return on physical capital, something which cannot be achieved by means of private expenditure in education only. When financed by a uniform income tax, public schools are also an instrument for intergenerational redistribution. Depending on initial conditions, the model predicts either a poverty trap (poor societies invest too little in education) or persistent growth driven by the accumulation of human capital. The introduction of public financed education shrinks the set of initial conditions leading to the poverty trap. I characterize the global dynamics of the model, which delivers a number of testable hypotheses on the relation between income growth, capital accumulation and the development of public education. Throughout the paper, I concentrate on specific functional forms allowing for a closed form solution, nevertheless, all the important results carry over to fairly general utility and production functions.

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1. Introduction

This paper explores the idea that the fundamental reason for the public financing of education is to solve a free-rider problem adversely affecting accumulation of capital and economic growth.

In a society in which physical and human capital are complementary factors of production, the owners of next period's capital stock have a vested interest in the level of human capital of the future *average* worker. Absent a credit market where young generations can borrow against their future labor earnings, the equilibrium level of private financing of education will be less than required by productive efficiency. Even with parental altruism, the amount of resources going to the school system are likely to be less than economic efficiency requires. This is because private agents cannot appropriate their contribution to the increased productivity of capital that an extra dollar spent on aggregate education entails.

While these observations apply to most forms of education and training, here I prefer to interpret the word 'school' as referring to primary and secondary education only. Available evidence (see e.g. Psacharopoulos (1985, 1989) and references therein) suggests this is where the social rate of return is at its highest level, and where public financing is most concentrated. I make no claim of originality for this interpretation of the role of schooling: scholars in the economics of education and in the human capital literature have long stressed the fact that the *main* purpose of a public school system is to provide society with an educated and skillful workforce (see e.g. Becker (1975), Butts (1978), Friedman (1962), and Stiglitz (1974), to name but just a few). My contribution amounts to proposing a dynamic general equilibrium model which formalizes this idea, and to investigate its theoretical implications. The ultimate purpose of the exercise is to develop a theory of the interactions between education and economic growth in a context in which markets are not complete and intergenerational linkages play a crucial role in the accumulation of capital. The research initiated here has been developed and pushed farther ahead in Boldrin and Montes (1997, in press), Boldrin and Rustichini (2000), Boldrin and Jones (2002, 2005), and Boldrin et al. (2004).

The basic structure is that of an OLG world where individuals live for three periods: young, middle age and old. When young they are endowed with a unit of time, which they can use either to go to school or for leisure. During their middle age, their unit of labor time is inelastically supplied to the productive sector. The income so obtained is divided between current consumption, saving (in the form of physical capital) and education of the children. When old, they consume the return on the accumulated stock of physical capital and then die. Production of the homogeneous good is described by a standard neoclassical production function $Y=F(K,H)$, which uses physical capital (K) and effective labor (H) (one unit of labor time multiplied the level of human capital) as inputs. The growth rate of human capital depends on the amount of time and physical resources devoted to it. When the latter are zero, human capital shrinks from one generation to the next, if anything because human capital is embodied in people, and people die; see Boldrin and Jones (2005) for an application of this idea to the emergence of the industrial revolution. Per capita human capital may, instead, grow if enough inputs are applied to the schooling process. I assume that, a positive growth rate of aggregate human capital can be obtained by investing a finite amount of resources. Question is, who has the incentive to carry out the appropriate amount of investment?

I find it reasonable to assume that young people attending school cannot borrow from the older generations. While some credit instruments to finance private investments in human capital do exist, they seem to concentrate on the latest stages of the training process. They are almost non-existent outside of the United States, where a large portion of higher education is publicly financed (but not necessarily publicly provided, see James (1992)). Furthermore, I am mostly concerned with primary and secondary education, for which these markets are almost never available (according to West (1991, p. 168), Japan is an important exception; see also Ehrlich and Lui (1991) for a different theoretical interpretation of this matter).

If young people cannot properly finance their education, who will? While the old owners of capital have little interest in future productivity, the middle age individuals have a *collective* interest in fostering society's stock of human capital. This can be achieved by instituting a publicly financed school system. Such a justification for public schooling does not depend on the assumption that the educational process generates positive external effects. Rather, it is the presence of factor complementarities in production and the absence of perfect capital markets that make a certain amount of public education socially efficient. As I am also interested in studying the interactions between private and public financing, I later extend the basic model to allow for parents that care about the level of education their children achieve. This is formalized by introducing the human capital of the young into the utility function of the middle aged.

Assuming the existence of a state with the power to collect taxes creates a distributional conflict between generations, as they hold opposite interests over the kind and amount of taxes that should be levied. I proceed under the assumption that such conflicts are solved by means of majority voting: the agents that are alive and have electoral rights vote on an income tax level and on the allocation of its revenues.

This leads to a number of strategic considerations, which here I circumvent by assuming a weak form of myopia in the voters' decision making process. Such issues are addressed, together with other related to the design of an intergenerational efficient and 'fair' welfare systems, in the joint papers with Montes and with Rustichini quoted earlier. In the present context, I assume that middle age individuals, while perfectly able to recognize the impact of taxation on the current consumption-saving decisions, ignore its indirect effect on the *future* tax rates. In other words they take next period tax rate as given and select their best response to it. I justify this assumption on the ground of tractability, but the discussion in Section 2 shows that the qualitative results would not change if I were to consider Markov Perfect Equilibria. The results of Boldrin and Montes (1997) and of Boldrin and Rustichini (2000), on the other hand, show that allowing for just sub-game perfectness greatly, and unreasonably, extends the number of possible equilibria.

In this model the median voter turns out to be the representative agent from the middle age generation who is in fact the only one facing a meaningful tradeoff when selecting a fiscal policy. Given the particular choice of production and utility functions, the global dynamics of the system can be explicitly computed by solving the middle age political optimization problem and feeding it back into the competitive equilibrium.

Exception made for the last section, I always assume the existence of a representative agent in each generation. Hence, I do not examine the intra-generational income distribution problems associated with voting on public education, which are instead the object of a number of recent studies, e.g. Eckstein and Zilcha (1994), Gloom and Ravikumar (1992), Perotti (1993), and Saint-Paul and Verdier (1993). Still a number of interesting issues in the theory of economic development can be addressed within the model I just described.

The process of economic growth involves the transferring of an ever growing stock of resources from the old to the young generations. In the representative agent infinite horizon model, this problem is solved at once by assuming a bequest motive that effectively transforms individuals into infinitely lived dynasties. If one believes the latter solution to be too simple, as I do, then one faces the problem of explaining persistent growth from within an OLG framework when a substantial portion of the productive resources are not bequeathed from one generation to the next. It is well known (Boldrin, 1992; Jones and Manuelli, 1992) that in this case persistent growth will not obtain if the technology is modeled as a one sector with constant returns to scale. A number of channels for transferring wealth from the old to the young generations have therefore been proposed. This paper models (private and public) provision of education as one such intergenerational wealth transfer.

A consequence of this hypothesis is that while the presence of parental altruism may lead one to believe that these transfers are fully voluntary, the political equilibrium I study suggests that a portion of them is most likely not. I show in fact that, even when privately financed schools are a viable option, a majority of voters may find it rational to maintain funding of education by means of income taxes as the latter (forcedly) transfer income from the old to the middle and young generations. Public schooling, therefore, is not just an instrument for intra-generational income distribution, but also one of the intergenerational income redistributions. The latter, I argue, may well be the key element guaranteeing a persistent increase in per-capita income.

The last statement is based on the idea that growth is due to the synergies between human and physical capital and to the absence of effective bounds on the accumulated stock of the first. This implies, though, that not only continuous accumulation but also the lack of it should be explainable from within our framework. A number of authors have developed models in which growth is due to the accumulation of human capital (Caballé and Santos, 1993; Lucas, 1988; Uzawa, 1965), but the question of the existence of a poverty trap seems to have received scarce attention. The problem has a simple solution in the context I study, where the existence of a poverty trap is independent of the presence of any kind of technological externality. It is related instead to the lack of initial income which renders the economic agents unwilling to invest in the future generations (either privately or by means of an income tax) and/or to its inadequate distribution which determines a wicked anti-growth alliance between the destitute part of the society and the old owners of the stock of capital.

The two reasons I adopt to justify the provision of education (altruism and increased productivity) are perfectly compatible in the sense that in equilibrium we should typically observe both private and public expenditure contributing to the accumulation of human capital. The model can be extended to deliver just that. I use it in section three to also show that the intergenerational transfer implicit in public financing may result in a 'crowding out' of private expenditure under certain circumstances. In this context I point out that, while at high-income levels a relatively inefficient public system will be replaced by private schools, the former may come into existence at very low-income levels and help the economy to jump-start its growth process. This could not occur if private financing were the only option.

In a static model of school expenditure Peltzman (1973) has argued that the public provision of education may result in an equilibrium with less total expenditure in schooling due to the fact that using the service provided by public institutions prevents families from appropriately supplementing it with private contributions. This is related to the important debate about the effects that a fiscally supported system of educational vouchers would have on the accumulation of human capital (see e.g. Friedman (1962)).

In Section 4, I add heterogeneous agents to the basic model and modify the technology to incorporate Peltzman's observation. This may have the unpleasant consequence of creating a multiplicity of political equilibria (Stiglitz (1974)), which I avoid by making a number of simplifying (but, I believe, empirically justifiable) assumptions. It turns out that, generally, not only the demand for education goes down but also the total amount of public financing diminishes, as the strategic considerations implicit in the political process generate a much smaller equilibrium tax rate. This happens because the impossibility of supplementing public funds with private ones lead the wealthier portion of the middle age generation to escape from the public system altogether. When this 'run to the suburbs' occurs, the only portion of public financing this group is willing to support is the one affecting the future return on the aggregate capital stock. By shifting downward the whole distribution of preferred tax rate this yields an equilibrium with less public financing of education. I believe this observation may be of some relevance for the current political debate about the impact that the introduction of vouchers would have on the quality of education received by the poorest group of our society.

The plan of the paper is the following. Section 2 introduces the basic model, studies its equilibria and characterizes their long-run evolution. Section 3 adds parental altruism and stresses the intergenerational redistributive aspect of the public school system as a further motivation for voters' support. The long-run relation between per-capita income and educational spending is emphasized. In Section 4, I look further into this issue along the lines of the previous paragraph. Section 5 concludes.

2. Voting on public financing of education

2.1. The basic model

I study an economy composed of overlapping generations of identical agents living for three periods. Each generation is composed of a continuum of individuals, and population growth is such that each generation is $1+n$ times the size of the previous one. At the beginning of period $t=0$ two generations are alive: the old people (of total size $1/(1+n)$) and the middle age ones (of size 1). A new generation of young individuals (of size $(1+n)$)

is then born. The old agents already own the initial stock of capital K_0, while the middle age are endowed with human capital H_0. These are the two factors of production. The young individuals can only spend their time at school to acquire human capital. When middle-age they will work and carry out consumption-saving decisions. When old they consume the return on their accumulated stock of physical capital.

For the time being I assume that the amount of time available to young people is inelastically dedicated to education and that no utility is derived from leisure activities. I also assume that individuals have identical skills and identical initial human capital, and that parents are completely selfish and do not care for the education of their children. These and other restrictions will be removed in the subsequent sections.

The technological possibilities of this society are described by two production functions, one for the homogeneous consumption-investment good and the other for the accumulation of human capital. The first has the standard Cobb–Douglas form:

$$Y_t = K_t^{\alpha} H_t^{1-\alpha}$$

while I write the second as:

$$h_{t+1} = \frac{(\varepsilon + z_t)^{\gamma}}{(1+n)} h_t$$

Capital letters are used to denote aggregate variables and lower case letters denote per-capita values. So $y_t = Y_t/(1+n)^t$ is income per-capita which is equal to $k_t^{\alpha} h_t^{1-\alpha}$. Most of the ensuing analysis will be carried out in per-capita units. The parameters ε and γ are both positive and less than one and z_t denotes the quantity of per-capita physical resources devoted to education.

Borrowing against future income from human capital is impossible. For the purpose of calculating equilibria I assume the utility function to be separable and logarithmic. Most results can be preserved by using separable and homothetic utility functions.

The life-cycle optimization problem for an agent born in period $t-1$ is then

$$U_{t-1} = \max\{\log c_t + \delta \log c_{t+1}\} \tag{2.1}$$

$$\text{subject to}: c_t + s_t = (1-\tau_t)\omega_t$$

$$c_{t+1} = (1-\tau_{t+1})\pi_{t+1} s_t = \tilde{\pi}_{t+1} s_t$$

where τ_t is the tax rate in place during period t, individual labor income is $\omega_t = \omega_t h_t$, w_t is the wage rate and $\tilde{\pi}_t$ is the period t net return on capital. The consumer's behavior is summarized by the following policies

$$c_t = \frac{1-\tau_t}{1+\delta}\omega_t$$

$$s_t = \frac{\delta(1-\tau_t)}{1+\delta}\omega_t$$

$$c_{t+1} = \frac{\delta(1-\tau_t)}{1+\delta}\omega_t \tilde{\pi}_{t+1}$$

Competitive equilibrium in the input and output markets, together with the fact that the total expenditure on schooling $((1+n)^t z_t)$ has to be financed by taxes on current income $(\tau_t Y_t)$ yield:

$$\omega_t = (1-\alpha)k_t^{\alpha} h_t^{1-\alpha}$$

$$\pi_t = \alpha k_t^{\alpha-1} h_k^{1-\alpha}$$

$$k_{t+1} = \frac{\delta(1-\tau_t)}{(1+\delta)(1+n)}(1-\alpha)k_t^{\alpha} h_t^{1-\alpha}$$

$$h_{t+1} = \frac{h_t}{(1+n)}(\varepsilon + \tau_t k_t^{\alpha} h_t^{1-\alpha})^{\gamma}$$

2.2. The political equilibrium

To close the model and move on to the study of its dynamic implications, we need to determine the level of taxation τ_t. This is done by voting on the tax rate: at the beginning of each period t all the entitled citizens cast their vote on the government's fiscal policy. The selected tax rate then gets implemented, consumption-saving decisions are made and the process repeats itself again and again in all the subsequent periods.

A mildly realistic interpretation of the model recommends treating the young generation as composed of individuals which have not yet entered college, and that, as such, do not participate in the political decision making process. The assumption is of no harm, as the same conclusions would be reached even if I allowed the young agents to exercise some political power. As for the middle age and the old individuals, I will assume they all have equal voting rights.

The tax rate collecting the majority of votes will be implemented. This still leaves open the question of how a rational agent should decide to cast its vote in an environment such as this. This requires making assumptions about the set of available actions, their impact on the aggregate state variables and the notion of equilibrium adopted by the representative voter.

These issues can be addressed by appealing to game theoretical arguments. In what I call the 'strategic voter' model, it is assumed that agents understand that their votes, by affecting the tax rate, will influence the future state of the economic system. This, in turn, will affect future agents decisions about fiscal policies. Rational voting on the part of utility maximizing agents, therefore involves taking into account not only the impact that current tax rates have on future state variables but also the indirect effect this has on future tax rates.

This argument is relevant only for the median voters, as all the others will understand that in equilibrium their opinions do not matter. In the model being examined, the median voter is the representative member of the middle age generation so it is his choice of an optimal strategy that needs to be formalized. Set

$$u_t(\tau_t,\tau_{t+1}) = \log((1-\tau_t)\omega_t - s_t(\tau_t,\tau_{t+1})) + \delta\log((1-\tau_{t+1})\pi_{t+1}(\tau_t,\tau_{t+1})s_t(\tau_t,\tau_{t+1}))$$

The equilibrium tax rates sequence is the perfect Nash equilibrium of a game involving an infinite number of players: the middle age generations alive in the periods $t=0, 1, \ldots$ The following 'infinitely nested' set of optimization problems formalizes the median voter decision in period t:

$$\max_{\tau_t} u_t(\tau_t, \tau_{t+1}) \tag{2.2}$$

$$\text{subject to}: \tau_{t+1} = \arg\max_{\tau_{t+1}} u_{t+1}(\tau_{t+1}, \tau_{t+2})$$
$$\text{subject to}: \tau_{t+2} = \arg\max_{\tau_{t+2}} u_{t+2}(\tau_{t+2}, \tau_{t+3})$$
$$\text{subject to}: \tau_{t+3} = \ldots \text{ etc. } \ldots$$

An equilibrium is then a sequence of functions $\tau_t^*(\cdot)$ such that $\tau_t = \tau_t^*(\cdot)$ solves (2.2) given $\{\tau_0^*(\cdot), \tau_1^*(\cdot), \ldots, \tau_{t-1}^*(\cdot), \tau_{t+1}^*(\cdot), \ldots\}$. The political equilibrium so obtained is a perfect Nash equilibrium along which each middle age generation chooses the tax rate τ_t optimally by fully discounting the effect it will have upon τ_{t+1} and so on.

In spite of its strong theoretical attributes, I will *not* make use of the strategic voter assumption to close the model. The motivation is, essentially, one of mathematical tractability: the hypothesis that each generation chooses its optimal strategy $\tau_t^*(\cdot)$ by taking into account the effects it will have on the value of $\tau_{t+1}^*(\cdot)$ makes it impossible (at least for me) to derive an analytical representation of the equilibrium sequence. The problem goes much deeper than a simple matter of computability. While the general results contained in Harris (1985) may be used to prove that a perfect Nash equilibrium exists for the game defined in (2.2), this is of little help for our purposes. To carry out a meaningful analysis we would require the existence of an equilibrium representable by means of a stationary, continuous function of the state variables. This is a difficult mathematical issue, akin to those studied in Bernheim and Ray (1985) and Leininger (1986), and upon which I would rather avoid dwelling in these circumstances. The literature on the topic is quite extensive, I will refer the reader to the recent discussion of this matter contained in Chari and Kehoe (1990), and to Krusell and Rios-Rull (1996) for an approach based on numerical computation. A few years after the first version of this paper was prepared, Boldrin and Montes (1997, in press), Boldrin and Rustichini (2000), and Forni (2005), did solve this problem, albeit partially, in models that grew out of the present one. I refer the readers to these articles for a more complete treatment of the problem.

Dropping the perfectness requisite is enough to deliver an analytically tractable notion of political equilibrium. In choosing its utility maximizing tax the median voter will simply take next period tax rate as a given number, and not as a function of the future state variables. In this way the maximization problem (2.2) collapses to the much simpler one

$$\max_{0\le\tau_t\le 1} u_t(\tau_t, \tau_{t+1}) \tag{2.3}$$

$$\text{subject to}: \tau_{t+1} \text{ given in } [0, 1]$$

the solution of which is, in general some time-invariant mapping τ^* representing the equilibrium tax τ_t as a function of the current state (K_t, H_t) and the future tax rate τ_{t+1}. An equilibrium is then a sequence of tax rates $\{\tau_t^*\}_{t=0}^{\infty}$ such that τ_t^* solves (2.3) given τ_{t+1}^*.

The equilibrium tax is again the one chosen by the representative individual in the middle age group. In fact an agent born in period $t-2$, who belongs to the old generation during period t, will cast his vote by solving:

$$\max_{0\leq\tau\leq 1} \log((1-\tau)\pi_t s_{t-1})$$

The solution to which is readily seen to be $\tau_t\equiv 0$. The old people have nothing to gain by investing in the education of the young generation: this only takes away current income from them and deliver a future increase in productivity, which they cannot enjoy. A middle age individual, on the other hand, faces a more interesting tradeoff: by giving up some income today it will enjoy a higher return on capital tomorrow. An agent born in period $t-1$, solves the following problem when voting during period t:

$$\max_{0\leq\tau_t\leq 1} \log((1-\tau_t)\omega_t - s(\tau_t)) + \delta \log((1-\tau_{t+1})\pi_{t+1}(\tau_t)s(\tau_t)) \tag{2.4}$$

The outcome would not change if we had allowed young people to vote: their preferred tax rate is always equal to one. The only borderline case left is the one in which there is no population growth and the young agents do not vote: then $\tau_t=0$ and the solution to (2.4) would each receive 50% of the vote and some ad hoc mechanism would have to be introduced to break the tie.

2.3. Equilibrium dynamics

Go back to the maximization problem (2.4). Making use of the competitive equilibrium values of s_t and π_{t+1} and massaging the first order conditions yield:

$$\tau_t^* = \frac{\gamma\delta(1-\alpha)}{\gamma\delta(1-\alpha)+(1+\alpha\delta)} - \frac{\varepsilon(1+\alpha\delta)}{(\gamma\delta(1-\alpha)+(1+\alpha\delta))y_t} \tag{2.5}$$

which after an obvious change of notation, can more parsimoniously be written as

$$\tau_t^* = \frac{a}{a+b} - \frac{\varepsilon b}{(a+b)y_t} \tag{2.6}$$

Consider now the implications of the voting rule (2.6) for the equilibrium dynamics. Notice first that a ‘poverty trap’ mechanism is always at work here. Whenever the current income level is not enough, or the marginal return to investing in education is low (from the view point of future capital holders) the approved tax will be zero. To help the intuition let us look at the equilibrium tax that obtains for a general, homothetic, utility function $u(c)+\delta u(c')$. This is

$$\tau = 1 - \frac{\pi(\tau)}{g(\pi(\tau))\cdot \partial\pi/\delta\tau} \tag{2.7}$$

where $g(\pi)$ is the function satisfying $s=(1-\tau)\omega g(\pi)$. Even in its general version the model predicts that public funding for education should be an increasing function of the level of investment in physical capital or (more precisely) of the portion of current income that is invested in the future capital stock. Moreover under the reasonable assumptions that $g(\pi)$ is elastic and $\pi(\tau)$ is a concave function, (2.7) also suggests that the tax rate should be a decreasing function of the expected return on investment. In situations where the return on physical capital is decreasing, the model predicts a higher willingness to accept taxes whose proceedings will be invested to foster accumulation of human capital.

On the other hand, a zero tax rate will be voted by the middle age group whenever the return on physical capital is too high relative to the amount of income allocated to it, i.e. whenever

$$\frac{\partial\pi/\partial\tau}{\pi} \leq \frac{1}{g(\pi)}$$

The latter inequality also suggests that, in a model with heterogeneous agents the distribution of wealth and savings among the members of the middle age generation should affect the political equilibrium. Middle age individuals with little or no physical capital would tend to oppose an income tax to foster education, whereas 'rich' people would most likely support it. This observation reveals a crucial interaction between the allocation of physical wealth and the level of public investment in education whose implications ought to be more fully investigated.

In our model the no-taxes condition becomes:

$$y_t \leq \frac{\varepsilon(1+\delta\alpha)}{\delta\gamma(1-\alpha)} = y_{\min} \tag{2.8}$$

which is much simpler than the one for the general case, but retains most of its qualitative implications. Given the parameters of the utility and production functions, (2.8) boils down to a simple restriction on income levels: poor countries tend to vote against financing of education, from which the poverty trap. Furthermore, countries with a low saving rate will be less likely to invest in public education, and countries where the physical stock of capital receives a larger portion of national income will require a higher level of income per-capita to invest in public education.

When the median voter favors a positive tax the per-capita amount of resources devoted to education turns out to be equal to

$$z_t = \tau_t y_t = \frac{a y_t}{a+b} - \frac{\varepsilon b}{a+b} \tag{2.9}$$

which is an increasing function of income as one would have expected. Plugging (2.9) into the law of motion of human capital delivers a simple growth condition

$$\left\{\frac{h_{t+1}}{h_t} > 1\right\} \Leftrightarrow \left\{y_t > \left(\frac{(a+b)(1+n)^{(1/\gamma)}}{a} - \varepsilon\right) \equiv \underline{y} > y_{\min}\right\} \tag{2.10}$$

which stresses the fact that some investment in education is not necessarily enough for growth. In order words the poverty trap extends to income level beyond those implied by (2.8).

The qualitative properties of the equilibrium dynamics can now be derived. For all pairs of initial conditions (h_t, k_t) such that $k_t^\alpha h_t^{1-\alpha} \leq y_{\min}$, one has

$$\begin{cases} k_{t+1} = \dfrac{\delta(1-\alpha)}{(1+\delta)(1+n)} k_t^\alpha h_t^{1-\alpha} \\ h_{t+1} = \dfrac{\varepsilon^\gamma}{1+n} h_t \end{cases} \tag{2.11}$$

while for pairs of initial conditions (h_t, k_t) such that $k_t^\alpha h_t^{1-\alpha} > y_{\min}$, the equilibrium dynamics is

$$\begin{cases} k_{t+1} = \dfrac{\delta(1-\alpha)b}{(1+\delta)(a+b)(1+n)} (\varepsilon + k_t^\alpha h_t^{1-\alpha}) \\ h_{t+1} = \left(\frac{a(\varepsilon + k_t^\alpha h_t^{1-\alpha})}{a+b}\right)^\gamma \dfrac{h_t}{1+n} \end{cases} \tag{2.12}$$

The dynamical system (2.11) has its only stationary state at the origin, which is obviously attracting for all orbits starting nearby. The system (2.12) has instead a unique interior stationary state that lies at the intersection between the curve

$$(k^*)^\alpha (h^*)^{1-\alpha} = \underline{y} \tag{2.13}$$

and

$$k^* = \frac{\delta b(1-\alpha)(1+n)^{(1-\gamma)/\gamma}}{a(1+\delta)} \tag{2.14}$$

where $\underline{y}$ is the minimum income value that allows for positive growth in per-capita human capital, derived in (2.10). The stable ($W^s(h^*,k^*)$) and unstable ($W^u(h^*,k^*)$) manifolds of the stationary point (h^*,k^*) defined by (2.13) and (2.14), can be easily derived with standard methods from dynamical systems theory. The stable manifold is particularly relevant as it defines the border between the poverty trap area, in which too little resources are invested in education and income per capita decreases, and the growth area in which the amount invested in education is high enough to allow for persistent increase in income.

2.4. Modeling school attendance rates

It is rather straightforward to replicate most of the previous results in a model where the members of the young generation are allowed to choose the amount of time they spend at school. Once again I will adopt a very simple functional form to allow for explicit calculations of the equilibrium values. To save on notation I will also set the growth rate of population $n=0$ and pretend that, in front of an electoral tie, the will of the middle generation is enforced.

I start by assuming that, when young, a member of this society may either attend school or work in some productive activity that utilizes unskilled labor and requires no stock of physical capital (newspaper delivering, land mowing, babysitting, etc.). Working in this kind of 'underground' economy pays a fixed wage rate β per unit of time, and I normalize β so that it is expressed in 'utils'. The life-time utility function of an individual born at $t-1$

can now be written as

$$U_{t-1} = \beta(1 - \ell_{t-1}) + \log c_t + \delta \log c_{t+1} \tag{2.15}$$

The description of the educational system needs to be modified accordingly to make the growth of human capital a function of the amount of the time invested in schooling. To keep the new model close to the initial one and tractable at the same time, the following functional form will be adopted:

$$h_t = h_{t-1}(\varepsilon + \ell_{t-1}^{\theta} z_{t-1})^{\gamma} \tag{2.16}$$

with the parameter θ restricted to $0<\theta\leq 1$. Maximization of (2.15) under the budget constraints in (2.1), the new constraint (2.16) and $0\leq \ell_{t-1}\leq 1$, yields consumption-saving policies identical to those of the previous subsection, while the school attendance rate is given implicitly by the first order condition

$$\frac{\beta}{1+\delta} = \frac{\gamma\theta z_{t-1}\ell_{t-1}^{\theta-1}}{\varepsilon + \ell_{t-1}^{\theta} z_{t-1}} \tag{2.17}$$

For values of θ strictly less that one, the latter implies that as long as expenditure on education is positive, school attendance is always positive and increasing until full participation is achieved at a level of expenditure per capita equal to

$$\bar{z} = \frac{\varepsilon\beta}{(1+\delta)\gamma\theta - \beta}$$

To derive a closed form expression for the school attendance level we restrict our attention to the special case $\theta=1$. In this case we have

$$\ell_{t-1} = \frac{\gamma(1+\delta)}{\beta} - \frac{\varepsilon}{z_{t-1}} \tag{2.18}$$

Notice that $\ell_{t-1}=0$ now becomes an equilibrium if public expenditure is too low. It is also a straightforward matter to verify that young agents still prefer a tax rate equal to 100%, so that the median voter is still the representative middle age individual.

By substituting (2.18) into the utility function (2.15), together with the usual consumption saving policies, and then solving the new version of the maximization problem (2.4) the equilibrium tax rate can be computed. It turns out to be always positive and, in fact, independent of the income level. Its value is

$$\tau^* = \frac{a}{a+b}$$

which corresponds to the maximum achievable tax rate under the exogenous school attendance regime of the previous subsections. Notice, though, that the poverty trap has not disappeared in this version of the model, as the level of school attendance will now be zero for all income levels satisfying

$$y_t \leq \frac{\varepsilon}{a}\left(b + \frac{\beta(a+b)}{\gamma(1+\delta)}\right)$$

More generally, the dynamical system describing the growth process when $\ell_t > 0$ is now

$$\begin{cases} k_{t+1} = \dfrac{\delta(1-\alpha)b}{(1+\delta)(a+b)}(k_t^\alpha h_t^{1-\alpha}) \\ h_{t+1} = \left(\frac{a\gamma(1+\delta)}{\beta(a+b)} k_t^\alpha h_t^{1-\alpha}\right)^\gamma h_t \end{cases} \tag{2.19}$$

which exhibits qualitative properties that are completely analogous to those of (2.12). In particular, for all initial conditions (h_t, k_t) such that

$$k_t^\alpha h_t^{1-\alpha} < \frac{\beta(a+b)}{a\gamma(1+\delta)}$$

the growth rate of the per-capita human capital stock will be less than one, while the opposite is true when the latter inequality is violated.

2.5. Taking stocks

We can summarize the predictions of the simple model introduced in this section along the following lines. For a given level of average education, support for public financing of schools will appear when the stock of physical capital reaches a critical level, before which persistent accumulation of human capital will not be observed. The portion of national income devoted to public education increases with income, but is bounded above by some number less than one. The same is true of the school attendance rate among young individuals. The latter can also be kept near zero if the amount of resources devoted to education is inadequate.

The prediction that both taxes and total amount of resources devoted to education go up when income per-capita increases, should be contrasted with that of models where public education is motivated only by intra-generational redistribution purposes. In those cases the portion of income devoted to public education decreases when average income increases, whereas the total amount of resources allocated to the school system may go either way. Casual evidence seems to suggest the opposite is true, both across countries and over time.

3. Parental altruism

In this section, I test the performances of the basic model when parents are assumed to be altruistic, thereby providing a second motivation for the provision of schooling. Indeed the introduction of altruism is a necessary requirement for studying the relation between public and private school financing, as the latter seems to be understandable only on the ground of parental generosity. The introduction of altruism in general capital accumulation models can, by itself, explain the existence of schooling and the persistence of growth. Still it leaves as unexplained the widespread adoption of publicly supported schools as an instrument for increasing society's average human capital, something which we capture in our framework.

To avoid collapsing the model into one of an infinitely lived dynasty I will assume that parental altruism expresses itself in the form

$$U_{t-1} = \log c_t + \delta \log c_{t+1} + \log h_{t+1} \tag{3.1}$$

Parents care about their children well being only insofar as this has to do with their education. They will provide for schooling, but will not leave any other kind of physical bequests to their offsprings.

3.1. Private versus public school financing.

Denote with z_t^p the portion of income that parents will be privately willing to devote to the education of their children. The total amount of per-capita resources available for human capital accumulation is then equal to

$$z_t = z_t^p + \lambda \tau_t y_t$$

where $\lambda = \lambda(\tau)$ takes values in the unit interval and is a meant to capture administrative costs and other factors making public financing relatively more inefficient than private financing. While no definite conclusion seems to have been reached on this point, the available evidence suggests that, net of the deadweight losses of taxation, the empirical value of λ may be pretty close to one (Levin, 1991; West, 1991). The main point of this section only requires the assumption that $\lambda(\tau)$ is a non-increasing, continuously differentiable function. To keep things explicit I will consider two particular functional forms: $\lambda(\tau) = \lambda$ and $\lambda(\tau) = \lambda/(1+\tau)$.

It is also important to note that in this section, as in the previous one, I am still assuming that public education works under a voucher-type system and that actual provision of the service occurs through a competitive market. This allows parents to supplement public funds with private ones and justify the new definition of z_t. The case in which public education entails public provision of educational services will be considered in the next section.

To spare us the burden of an excessive notation set $\gamma = 1$ in the human capital accumulation rule, this will not affect the final result in any significant manner. When the maximization of (3.1) under the budget constraints $c_t + z_t^p + s_t \leq (1-\tau_t)\omega_t$ and $c_{t+1} \leq \tilde{\pi}_{t+1} s_t$, results in a set of interior solutions one has

$$\begin{aligned} c_t &= \frac{1}{2+\delta}((1-\tau_t)\omega_t + \varepsilon + \lambda \tau_t y_t), \\ s_t &= \frac{1}{2+\delta}((1-\tau_t)\omega_t + \varepsilon + \lambda \tau_t y_t), \\ z_t^p &= \frac{1}{2+\delta}((1-\tau_t)\omega_t - \varepsilon(1+\delta) - (1+\delta)\lambda \tau_t y_t) \end{aligned} \tag{3.2}$$

The latter formulas emphasize the fact that the middle age generation's current disposable income is now greater than its after-tax wage payments. It includes the real value of government transfers for education ($\lambda \tau_t y_t$) and the fixed value ε. One will also notice that the non-negativity constraint on z_t^p will be binding when the tax rate is too high and/or current labor income too low, i.e. when

$$\{z_t^p = 0\} \Leftrightarrow \left\{ \tau_t \geq \frac{1-\alpha}{(1-\alpha)+\lambda(1+\delta)} - \frac{\varepsilon(1+\delta)}{y_t((1-\alpha)+\lambda(1+\delta))} \equiv \bar{\tau}(y_t) \right\}$$

Notice first that a 'crowding out' mechanism has already been introduced by assuming that public and private funding have some degree of substitutability. Given current income, if taxes exceed the $\bar{\tau}(y)$ level, no private expenditure should be observed. We should stress that, within the context of the present model, the crowding-out of private expenditure has no negative implication whatsoever on social welfare. It simply suggests that, holding income levels constant, we should observe less private spending on education in countries where public support is stronger.

The model still predicts that a poverty trap will be present, as the private investment in education may be zero even when the tax rate is zero. As before, this will occur when the country is too poor and/or when the share of income going to capital owners is very low, i.e. when

$$y_t \leq \varepsilon(1+\delta)/(1-\alpha) = y^1$$

We should determine next the equilibrium level of taxation when $y_t \geq y^1$. I will assume the median voter takes into account the non-negativity constraint on z^p when choosing his tax rate, i.e. he chooses the value of $\tau \in [0, \bar{\tau}(y)]$ that maximizes

$$\begin{aligned}
&\log\left(\frac{(1-\tau)\omega + \varepsilon + \lambda(\tau)\tau y}{2+\delta}\right) \\
&\quad + \delta \log\left(\frac{\delta}{2+\delta}\tilde{\pi}(\tau)((1-\tau)\omega + \varepsilon + \lambda(\tau)\tau y)\right) \\
&\quad + \log\left(h\left(\varepsilon + \lambda(\tau)\tau y + \frac{(1-\tau)\omega - \varepsilon(1+\delta) - \lambda(\tau)(1+\delta)\tau y}{2+\delta}\right)\right) \qquad (3.3)
\end{aligned}$$

The first-order conditions characterizing an interior solution are:

$$\lambda(\tau) + \tau\lambda'(\tau) = 1 - \alpha \qquad (3.4)$$

The intergenerational redistribution of income achieved by means of taxation, together with the relative degree of inefficiency of the public financing system become the crucial factors in the political decision process. Eq. (3.4) says that when choosing the portion of publicly supported educational expenditures the median voter will, on the margin, balance the tradeoff between income redistribution and efficiency losses. For the two functional forms of $\lambda(\tau)$ we have chosen, the equilibrium tax rate turns out to be:

$$\lambda(\tau) = \lambda: \quad \tau_t^* = \begin{cases} 0, & \text{if } \lambda + \alpha < 1 \\ \bar{\tau}(y_t), & \text{if } \lambda + \alpha > 1 \end{cases}$$

$$\lambda(\tau) = \frac{\lambda}{1+\tau}: \quad \tau_t^* = \begin{cases} 0, & \text{if } \lambda + \alpha < 1 \\ \sqrt{\dfrac{\lambda}{1-\alpha}} - 1, & \text{if } \lambda + \alpha > 1 \end{cases}$$

The dynamic implications of this result can be derived, by adapting the logic already used in the previous section. Under either one of the possible regimes (i.e. $\tau = 0$ or $\tau = \bar{\tau}$ or $\tau\sqrt{\lambda/(1-\alpha)} - 1$) there still exists some lower bound delimiting the set of initial conditions for which persistent accumulation is an equilibrium outcome. All the remaining qualitative predictions I have listed at the end of section two, remain true after the introduction of altruistic parents.

In particular it remains true that the amount of public resources devoted to education increases with income. When the efficiency loss is modeled in the form $\lambda/(1+\tau)$ the model also predicts that the ratio between private and public expenditures $z_t^p/\tau_t y_t$ will increase when income per-capita increases.

3.2. Public schools as shoe laces

In this subsection, I will argue that, even if relatively inefficient, public financing of education may be conducive of aggregate growth in situations in which the private altruistic motive would not suffice to deliver it.

To show this I impose the restriction $\lambda + \alpha < 1$, so that no public financing of schools would emerge in the political equilibrium if the income level were high enough to induce some positive private spending on education. Remember next that, for any given level of $\tau_t \geq 0$, private expenditure will be zero when the income level is below $\varepsilon(1+\delta)/(1-\alpha) = y^1$. In these circumstances the optimization problem faced by the median voter at time t is quite different from (3.3). In particular, a rational voter will realize that no one-to-one offset occurs between an increase in public and a decrease in private expenditure on schooling, for the simple reason that the latter is already equal to zero. The median voter then maximizes the following objective function

$$\log\left(\frac{(1-\alpha)(1-\tau_t)y_t}{1+\delta}\right) + \delta \log\left(\frac{\tilde{\pi}_{t+1}(\tau_t)\delta(1-\alpha)(1-\tau_t)y_t}{1+\delta}\right) + \log(h_t(\varepsilon + \lambda(\tau)\tau_t y_t)) \tag{3.5}$$

Expression (3.5) is an increasing function of τ at $\tau = 0$ when

$$y_t > \frac{\varepsilon(1+\alpha\delta)}{(1+\delta(1-\alpha))\lambda} = y^2 \tag{3.6}$$

Now the latter is smaller than y_1 whenever λ and δ are close enough to one; in fact $\lambda > (1-\alpha)/[1+\delta(1-\alpha)]$ is sufficient, which we will assume in this subsection. In these circumstances we have

$$\lambda(\tau) = \lambda \Rightarrow \tau_t^* = \frac{1+\delta(1-\alpha)}{2+\delta} - \frac{\varepsilon(1+\alpha\delta)}{\lambda y_t(2+\delta)} \tag{3.7}$$

$$\lambda(\tau) = \lambda/(1+\alpha) \Rightarrow \tau_t^* = \frac{\lambda y[1+\delta(1-\tau)] - \varepsilon(1+\alpha\delta)}{\varepsilon(1+\alpha\delta) + (2+\delta)\lambda y} \tag{3.8}$$

If the public financing system is not utterly inefficient and the future is not discounted too heavily, it pays for the middle age median voter to support public schooling even when he would not be willing to afford any amount of private expenditure on education. From this we should conclude that, when private altruism is not enough, a very poor country may still pull itself up by its own shoe laces by taxing national income according to (3.7) or (3.8) and investing the revenues in the production of human capital.

Quite obviously the driving force behind this result is the intergenerational transfer of wealth that public schooling induces from the old owners of capital stock to the younger parents. It may be worth noticing again that also in this case, it is the intergenerational redistributive aspect that counts: had public financing been only an instrument for intra-generational redistribution it would have not turned out useful to compensate for an insufficient amount of parental altruism. This follows from the fact that education is a normal good and that by redistributing income from the richest to the poorest portion of a society one cannot manage to increase the *maximum* level of per-capita income.

To complete our argument we need to check that the amount of resources collected through (3.7) and (3.8) is enough to expand the stock of per-capita human capital. As the algebra becomes rather cumbersome when (3.8) is used I will examine here only the case in which the inefficiency factor λ is a constant and the equilibrium tax rate is determined by (3.7). Plugging the latter in the laws of motion for h_t and k_t gives:

$$\begin{cases} k_{t+1} = \dfrac{\gamma\varepsilon}{\lambda} + \gamma(k_t^\alpha h_t^{1-\alpha}) \\ h_{t+1} = \theta(k_t^\alpha h_t^{1-\alpha})h_t \end{cases} \tag{3.9}$$

where we have set

$$\gamma = \frac{\delta(1-\alpha)(1+\alpha\delta)}{(1+\delta)(2+\delta)}; \quad \theta = \frac{1+\delta(1-\alpha)}{2+\delta}$$

Straightforward algebraic manipulation reveals that the system (3.9) has a stationary state (h^*,k^*) at which

$$(k^*)^\alpha (h^*)^{1-\alpha} = \frac{1-\theta\varepsilon}{\lambda\theta} = y^3$$

holds. It is also easy to verify that for values of ε below 1 the double inequality

$$y^2 < y^3 < y^1$$

is satisfied, confirming that the case we are studying is not vacuous. The structure of (3.9) in a neighborhood of (h^*,k^*) is that of a saddle point and the global behavior of the system does not differ from the one derived previously. Also in this case it is the stable manifold of the stationary state that separates the poverty trap from the area in which perpetual accumulation occurs.

4. Heterogeneous agents prefer vouchers

I have already insisted on the fact that the 'public' nature of the system considered until now, follows from its source of financing and not from the way in which the service is provided. In fact a number of assumptions I have made (see e.g. Section 3) makes it resembles more an educational vouchers system financed by income taxes than the public school system we are familiar with. In this last section I move away from this restriction and introduce explicit assumptions aimed at capturing an important feature of publicly provided school services.

I concentrate my attention on the fact that provision of schooling involves a fundamental indivisibility: attending a school prevents a person from supplementing the educational services so obtained with services from another institution. More to the point, every school typically provides a fixed amount of education on a 'take-it-or-leave-it' basis. If more educational services are sought one would have to purchase *their totality* from a different source. While this technological restriction applies to private and public schools alike, the latter are characterized by the fact that it is very hard to increase/decrease the quality of education one receives by moving from one public school to another. Within a given school district a substantial uniformity exists and to move away from a district often involves very high transaction costs.

It has been observed (e.g. Peltzman (1973)) that such a mechanism tends to lower the total amount of education demanded relative to a system (such as the one I considered earlier) in which the government is transferring educational dollars to families which eventually purchase school services in a competitive market.

I will argue here that in a dynamic setting such as the one I have illustrated in this paper, public provision of schooling tends to lower also the amount of funding allocated to public education and consequently slows down the process of capital accumulation. The intuitive reason for this outcome is that, given a certain amount of publicly provided education there will always be families who are receiving less than they consider optimal. If these families were allowed to supplement the government funding with private ones, they would simply do so and nothing essential would change. When this is either impossible or very costly, those parents seeking a high level of expenditure on the education of their children will be forced to give up the whole amount of funding coming from public sources and bear the *full* cost of private education. From the point of view of these individuals the amount paid in taxes is deprived of most of the utility they would otherwise have attached to it. When it comes time for voting they will be willing to support either a much higher or a much smaller tax rate: under the first they will demand only public school services, while in the second case they will continue to use the private sector.

While simple to state, this mechanism is hard to analyze as it involves a number of strategic subtleties that (as pointed out by Stiglitz (1974)) may easily lead to a plethora of different equilibria. As I am not interested in this line of reasoning, I will force the argument through by exploiting a number of simplifying (but not necessarily unrealistic) features of the functional forms I have chosen.

Assume that the growth rate of the population is negligible: this implies that, in order to be approved, a positive tax rate should receive the support of the near totality of the members of the middle age group. While this does not change the substance of the analysis it simplifies the task of characterizing the equilibrium tax rate. This is in fact going to be the *smallest* level of taxation supported by a member of the middle age generation: all the old individuals still cast their vote against the tax and all the young ones still would like to see a tax equal to the maximum allowed.

In this context the pivotal voter becomes the one who would choose the public system if the latter could provide him with a very high level of services, but that would opt for a private school otherwise. This reduces the set of possible equilibria to the following two classes: one in which everybody takes advantage of the public system, and a second in which a certain portion of the population shifts to the private system. I claim that the second equilibrium will obtain whenever the initial distribution of human capital crosses a critical threshold level of dispersion. I also show that, contrary to what would be true in the voucher-model of sections two and three if heterogeneous agents were introduced there, the median voter belongs now to the upper tail of the income distribution curve. This conclusion goes against the general wisdom of most models in which public education is motivated by the desire of the poorest portion of the population to transfer some income away from the members of the richest segment. It also suggests that the current methods of public school provision may actually go against their claimed redistributive purposes.

Assume that agents are heterogeneous in human capital levels, and impose the 'either public or private' restriction on the educational technology. Assume then that each generation is still composed of a continuum of agents of size $(1+n)^t$, with type $i\in[0,1]$ reproducing itself from one generation to the next at a uniform rate $1+n$. The assumption that parents care about their children only insofar as their human capital is concerned but do not leave physical bequests, turns out to be very useful in this context as it breaks down the intergenerational linkage due to the physical stock of capital. Its presence would have enormously complicated our analysis and is better left for future considerations.

As I do not plan to examine the dynamic evolution of the allocation of income and human capital, I will make no special assumptions on the initial distribution of types $\mu(i)$. The aggregate state variables are defined as:

$$k_t = \int_0^1 k_t^i \mu(\mathrm{d}i); \quad h_t \int_0^1 h_t^i \mu(\mathrm{d}i); \quad y_t = \left(\int_0^1 k_t^i \mu(\mathrm{d}i)\right)^{\alpha} \left(\int_0^1 h_t^i \mu(\mathrm{d}i)\right)^{1-\alpha} = k_t^{\alpha} h_t^{1-\alpha}$$

The life-cycle optimization problem of an individual of type i, born in period $t-1$ is now

$$\max\{\log c_t^i + \delta \log c_{t+1}^i + \log h_{t+1}^i\}$$

$$\begin{aligned} &\text{subject to : } c_t^i + s_t^i + z_t^{ip} \le (1-\tau_t)\omega_t^i \\ &c_{t+1}^i \le \tilde{\pi}_{t+1} s_t^i \\ &h_{t+1}^i = h_t^i(\max\{h(z_t^{ip}), h(z_t)\}) \end{aligned} \tag{4.1}$$

where $h(x)=(\varepsilon+x)^{\gamma}$, and $z_t=(\tau_t y_t)/m_t$, with $m_t\in[0,1]$ denoting the equilibrium portion of the population attending public schools. The 'max' in the law of motion for human capital captures the exclusionary mechanism I discussed earlier. I have dropped the hypothesis of 'relative inefficiency' of public schools to save us some notation.

Manipulation of the first order conditions yields $s_t^i = \delta c_t^i$ as usual. The individual demand for private education requires a more detailed analysis. Begin by observing that given the common preferences and the fact that education is a normal good, those individuals demanding a higher total level of educational spending are also the wealthiest among the members of the middle age group. Given m_t and τ_t and by re-ordering types so that higher indices always correspond to higher incomes we have:

$$z_t^{ip} = 0 \Leftrightarrow \frac{h'(z_t)}{h(z_t)} \leq \frac{1}{(1-\tau_t)\omega_t^i - s_t^i - z_t} \quad \text{for } i \in [0, m_t] \tag{4.2}$$

and

$$z_t^{ip} > z_t \Leftrightarrow \frac{h'(z_t^{ip})}{h(z_t^{ip})} = \frac{1}{(1-\tau_t)\omega_t^i - s_t^i - z_t^{ip}} \quad \text{for } i \in [m_t, 1] \tag{4.3}$$

The curious consumption pattern implied by (4.2) and (4.3) and the ‘take-it-or-leave-it’ provision should be stressed here. Under appropriately chosen parameter values there will exist an intermediate group of agents (the ‘not so rich’ among those choosing to buy private education) that will have a consumption level lower than those individuals immediately below them in the income scale that are opting for the public school system. Personal introspection, even 12 years later and with a son now attending college, supports the model’s prediction.

Given a pair (m_t, τ_t), individual consumption-saving rules are the following. *Public School Families*, $i \in [0, m_t]$:

$$c_t^i = \frac{1-\tau_t}{1+\delta}\omega_t^i$$

$$s_t^i = \frac{\delta(1-\tau_t)}{1+\delta}\omega_t^i$$

$$c_{t+1}^i = \frac{\tilde{\pi}_{t+1}\delta(1-\tau_t)}{1+\delta}\omega_t^i$$

Private School Families, $i \in [m_t, 1]$:

$$c_t^i = \frac{(1-\tau_t)\omega_t^i + \varepsilon}{1+\delta+\gamma}$$

$$s_t^i = \frac{\delta((1-\tau_t)\omega_t^i + \varepsilon)}{1+\delta+\gamma}$$

$$z_t^{ip} = \frac{\gamma(1-\tau_t)\omega_t^i - \varepsilon(1+\delta)}{1+\delta+\gamma}$$

$$c_{t+1}^i = \frac{\tilde{\pi}_{t+1}\delta((1-\tau_t)\omega_t^i + \varepsilon)}{1+\delta+\gamma}$$

The equilibrium levels of m_t and τ_t need to be determined simultaneously. Consider first the possibility of an equilibrium where $m_t=1$. The tax rate will then emerge from the following maximization problem

$$\max_{0\leq\tau\leq 1} \log\left(\frac{(1-\tau)\omega^i}{1+\delta}\right) + \delta\log\left(\frac{(1-\tau)\delta\tilde{\pi}\omega^i}{1+\delta}\right) + \log(h^i(\varepsilon+\tau y)\gamma)$$

for $i\in[0,1]$, which yields the following first-order condition for an interior solution

$$\frac{1+\delta}{1-\tau} = \delta\frac{\partial\pi}{\partial\tau}\frac{1}{\pi(\tau)} + \frac{\gamma y}{\varepsilon+\tau y} \tag{4.4}$$

Denote the unique solution to (4.4) with τ_t^*. Note that our choice of utility function makes it independent of the individual type: at the 'good' equilibrium there is unanimity about the level of taxation. This will not be true for more general utility functions. From (4.2) an equilibrium pair (m_t,τ_t) has also to satisfy:

$$m_t \leq \frac{(1+\gamma)\tau_t y_t}{\gamma(1-\tau_t)\omega_t^i - \gamma s_t^i - \varepsilon} \tag{4.5}$$

for all indices $i\in[0,m_t]$. Hence $(1,\tau_t^*)$ is *not* an equilibrium whenever there exists an agent $j\in[0,1]$ for which

$$\omega_t^j > \frac{(1+\delta)}{\gamma(1-\tau_t^*)}(\varepsilon+(1+\gamma)\tau_t^* y_t) \tag{4.6}$$

The right-hand side of (4.6) defines a threshold level of per-capita income, above which individuals will not be satisfied anymore with an equilibrium in which only public schools exist. Let us assume that the threshold defined in (4.6) is violated for some non-negligible measure ν_t of agents in [0,1].

When this is true all voters will realize that $(1,\tau_t^*)$, with τ_t^* determined from (4.4) cannot be an equilibrium. The political equilibrium will be determined by a new pair (m_t, τ_t), where $m_t<1$ satisfies (4.5) and τ_t is the smallest among the tax rates demanded by members of the middle age group. To characterize it we should examine the new voting problems they face. Take the proportions m_t and $\nu_t=1-m_t$ of public and private school users as given. An individual $i\in[0,m_t]$ will vote according to

$$\frac{1+\delta}{1-\tau} = \delta\frac{\partial\pi}{\partial\tau}\frac{1}{\pi(\tau)} + \frac{\gamma y}{\varepsilon m_t+\tau y} \tag{4.7}$$

which under our assumptions, still has a unique solution $\bar{\tau}(m_t)>\tau_t^*$. An individual $j\in[m_t,1]$ instead chooses his vote according to

$$\max_{0\leq\tau\leq 1} \log\left(\frac{(1-\tau)\omega^j+\varepsilon}{1+\delta+\gamma}\right) + \delta\log\left(\frac{\tilde{\pi}\delta((1-\tau)\omega^j+\varepsilon}{1+\delta+\gamma}\right)$$
$$+\log\left(h^j\left(\varepsilon+\frac{\gamma(1-\tau)\omega^j-\varepsilon(1+\delta)}{1+\delta+\gamma}\right)^\gamma\right)$$

which yields the first order condition

$$\frac{(1+\delta+\gamma)\omega^j}{(1-\tau)\omega^j+\varepsilon} = \delta\frac{\partial\pi}{\partial\tau}\frac{1}{\pi(\tau)} \tag{4.8}$$

A comparison of (4.7) and (4.8) shows that the solution to the latter will be strictly smaller than the solution to the former for some large enough level of income dispersion, i.e. for some appropriately chosen

$$\omega^j > \frac{\varepsilon(1+\delta)}{\gamma(1-\tau_t)}$$

The equilibrium tax rate will therefore be established around the level proposed by the richest portion of the population, which is *smaller* than the one proposed by the other members of the middle age group. As for the participation rate m_t notice first that (4.8) determines τ_t independently from m_t. Denote with i the richest among the agents voting according to (4.7) and with j the poorest among those voting according to (4.8) and let τ be the unique tax rate solving (4.8). In equilibrium we must have:

$$\frac{(1+\gamma)\tau y}{\gamma((1-\tau)\omega^j - s^j)-\varepsilon} \le m \le \frac{(1+\gamma)\tau y}{\gamma(1-\tau)\omega^i - \gamma s^i - \varepsilon} \tag{4.9}$$

In order for the latter to yield a unique value of m_t, one needs to assume a continuous distribution of income, which might be somewhat restrictive. The non-uniqueness which arises in the general case is, nevertheless, not particularly serious because one can always select the upper limit in (4.9) as the equilibrium value of m_t. As one would expect the latter is, in any case, an increasing function of the tax rate.

The equilibrium we have described seems to share a number of features often observed in the real world. Foremost among them is the fact that the richest portion of the population supports a lower level of public school financing than the poorest one, *and* that the median voter seems closer to the former than to the latter type of individuals. It also suggests, coherently with casual observation, that the support for public school financing and provision increases when income inequality decreases.

An important implication of the former observation is the following: when growth in average income is accompanied (as it seems to be in the real world) by a reduction of income inequalities, we should observe a correlation between increases in per-capita income and the amount of public financing of education. This appears to be consistent with the empirical work of James (1992) who observes more private schools in poor countries. Furthermore, if higher growth rates are (at least partially) the outcome of a higher investment in education then less inequality means more economic growth.

The implications for the dynamic behavior of our model economy is straightforward: under the ‘subsidy-in-kind’ system the equilibrium amount of public education provided is strictly less than the one that obtains under a voucher system. This impacts negatively on the overall process of human and physical capital accumulation which in turn leads to a slower growth rate of national income.

It is worth stressing that, while the arithmetics of the previous argument is greatly simplified by my choice of utility and production functions, the crucial point would be preserved by more general functional forms. The empirical relevance of the phenomenon I have pointed out and its actual impact on the growth process of our economies may be quantified by making appropriate use of the model developed here.

The general result seems to be quite consistent with what we observe in reality and provides strength to the argument claiming that the adoption of a market approach to the *provision* of education will in fact increase the equilibrium amount of resources devoted to it.

In fact one may push the argument further and claim that the introduction of monetary subsidies to education and the opening of a competitive market on the supply side, may help reduce the high amount of segregation we observe in many American communities. If we allow ourselves the freedom to extend the model outside its present boundaries, the first best for the rich portion of the population would be to try to create neighborhoods segregated along income lines with school financing to be provided locally. Something, indeed, we seem to observe extensively in the United States and which is the object of a recent study by Fernandez and Rogerson (1992).

While I am unable to deliver a precise result with respect to the dynamic evolution of the distribution of income, I find it reasonable to conjecture that one should observe a larger increase in income inequalities under the 'subsidy-in-kind' system than under the vouchers system considered in earlier sections.

5. Conclusions

I have proposed a model of schooling based on the fundamental idea that publicly subsidized education solves a free-rider problem in economies in which markets for financing of human capital investment are lacking. If human capital accumulation is one of the engines of growth, then public schools will tend to foster growth and will be introduced in those economies that have a high enough stock of physical capital to make the investment in education affordable and profitable at the same time.

When the amount of resources devoted to public education is decided by majority voting it becomes unavoidable to use it also as an instrument for intergenerational income redistribution. In my model, though, income redistribution runs from grandparents to children while the parents stand in between, equalizing marginal costs and marginal gains. This aspect becomes even more important when parental altruism is introduced: parents will then finance some of their children education by taxing the grandparents' income. The incentive to do so is reduced, or even eliminated, when the public system is particularly inefficient relative to the private one and when the portion of income going to the elderly owners of the stock of capital is small.

Nevertheless there exist circumstances under which even an inefficient public school system may be useful to bootstrap economic development: in a political equilibrium with majority voting, public financing of schooling may be introduced when private financing would not emerge in equilibrium. This transferring of resources to education may be enough to start a growth process which by increasing income per-capita past a critical level, may eventually lead to the dismissal of the public system in favor of a (supposedly more efficient) private one.

I have also argued that public provision of schooling, when practiced in the 'take-it-orleave-it' form which is the rule almost everywhere, will cause a decrease of the equilibrium amount of resources devoted to public education and a run away from it and toward private schools on the part of the richest segments of the population. This, in turn will result in a reduction of the aggregate growth rates of human and physical stocks of capital. The implications it may also have on the income distribution dynamics ought to be investigated further.

Finally, this paper is mute with regard to one important issue: do ut des. That is to say, can we conceive of social arrangements in which older generations, which have previously financed the education of their offsprings, may collect the return from this investment not just indirectly, via the increased return of their stock of physical capital, but also directly, i.e. by receiving a transfer from their working children in proportion to the amount of educational services they earlier provided them with? Public pensions are a mechanism to transfer income from the middle age to the old, and Boldrin and Rustichini (2000) show that such a system can indeed be sustained in a dynamic political equilibrium. The next question is, then, can an appropriately designed intergenerational welfare state replicate the allocation that would obtain if markets were complete, so that the middle age people would lend to the young via a bond market for education financing, and then receive interest and principal from this investment in their later ages? Boldrin and Montes (1997, in press) argue that this indeed is possible by properly designing the linkage between public education and public pensions, so that the complete market allocation is implemented in the ensuing competitive equilibrium with an efficient intergenerational welfare state.

6. Uncited references

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