Labor Contracts and Business Cycles

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This paper investigates the claim, often put forth by real business cycle proponents, that the poor performance of their models in matching real-world aggregate labor market behavior is due to the fact that observed real wage payments do not correspond to the actual marginal productivity of labor but contain an insurance component that cannot be accounted for by the Walrasian pricing mechanism. To test this idea, we dispense with the Walrasian description of the labor market and introduce contractual arrangements between employees and employers. Assuming that employees are prevented from accessing capital markets and are more risk averse than employers, we use the theory of optimal contracts to derive an equilibrium relation between aggregate states of the economy and wage-labor outcomes. This contractual arrangement is then embedded into a standard one-sector, stochastic neoclassical growth model in order to look at the business cycle implications of the contractual hypothesis. The resulting dynamic equilibrium relations are then parameterized and studied by means of standard numerical approximation techniques. The quantitative properties of our model appear to be somewhat encouraging. We have examined different contractual environments, and in all circumstances the contracts-based equilibrium performs better than standard ones with regard to the

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labor market variables and at least as well with regard to the other aggregate macroeconomic variables. The present paper reports only the simulation results relative to what we consider the most empirically relevant cases.

I. Introduction

Our point of departure is the observation that standard real business cycle (RBC) models perform poorly in mimicking the statistical properties of labor market fluctuations, factor share cyclical behavior, and the comovements between capital income share and investment variations. These are not particularly new remarks. Beginning with Summers (1986), a number of different authors have either dismissed RBC models because of this feature or tried to amend them.[1]

While investigators have maintained very different opinions about the appropriate framework capable of modeling the labor market's cyclical oscillations, there seems to be wide agreement on the stylized facts and on their inconsistency with the marginal productivity and intertemporal substitution models of the labor market.

Observed real wages are too smooth and estimated intertemporal labor supply elasticities too low to justify the observed volatility in hours. If (as the RBC models assume) employment and real wages are generated mainly by the impact of labor demand shocks on a competitive labor market, then the data should lie close to a dynamic labor supply function. If this supply function is inelastic, the variations in real wages should be larger than the variations in employment. Reality is orthogonal to the model's predictions.

Table 1 illustrates some features of the post–Second World War period for the U.S. economy. We have reported sample statistics on standard deviations, output correlations, and unconditional first autocorrelations for Hodrick-Prescott filtered data. While the adoption of different stationarity-inducing methods seems to affect the output-correlation and autocorrelation properties of certain time series, it is beyond the scope of this paper to address these differences. Since Hodrick-Prescott filtering is the method most often used to induce stationarity in the RBC literature, we report all statistics based on Hodrick-Prescott filtered data. Where applicable, we note differences

[1] A few of the latter include Hansen (1985), Prescott (1986), Rogerson (1988), Wright (1988), Blanchard and Fischer (1989), Aiyagari, Christiano, and Eichenbaum (1990), Christiano and Eichenbaum (1990), Benhabib, Rogerson, and Wright (1991), Danthine and Donaldson (1992), Rotemberg and Woodford (1992), Burnside, Eichenbaum, and Rebelo (1993), and Gomme and Greenwood (in press).

TABLE 1

Quarterly U.S. Data (1947:1–1990:4)

Series	Standard Deviation	Correlation	Autocorrelation
Output	2.24	1.00	.847
Consumption	.86	.75	.817
Investment	4.40	.81	.806
Hours	1.88	.88	.887
Average labor product	1.06	.55	.680
Real wage	.77	.33	.684
Labor share	1.08	−.32	.723
Profits	10.49	.81	.786

Note.—Correlation refers to the sample correlation with output. Autocorrelation refers to the sample unconditional first autocorrelation. Statistics are based on time series that have been filtered with the Hodrick-Prescott filter to assure stationarity. The Hodrick-Prescott filter was computed for $\lambda = 1{,}600$.

in results obtained from alternate methods: log-linear detrending and log first-differencing.

A few "facts" stand out quite clearly. Real wages exhibit a weak correlation with output and about half its volatility. Sample estimates also show that while in the long run wages and labor productivity may display a high degree of conformity, they do not exhibit much of a coherent relationship at business cycle frequencies. Furthermore, real wages are highly persistent, a property that is not shared by the real wage time series generated by the standard RBC model. Indeed, a high autocorrelation level is displayed by most aggregate variables in log first-differences as well (not reported in table 1). This is a crucial property of real business cycles that is seriously missed by standard RBC models.

Labor hours (and employment as well) are strongly procyclical and substantially more volatile than wages. In fact, depending on sample subperiods, they may display even wider oscillations than output itself. The very high elasticity of the dynamic labor supply curve "implied" by the aggregate data is at odds with most microeconomic evidence on labor supply behavior and is the crucial reason for the rejection of the intertemporal substitution model (Altonji and Ashenfelter [1980] and Altonji [1982] contain the seminal empirical work in this direction).

Analyses of micro-level data (as reported, e.g., in Beaudry and DiNardo [1991] and Bils [1991]) also reveal that wages depend on labor market conditions at the time workers are hired and that real wages are quite sensitive to variations in the unemployment rates that occur during the job tenure period.

Finally, it has long been observed that a high degree of coherence exists between most measures of profits and investment activity, with the former somewhat leading the latter (Zarnowitz 1992, chap. 2). Profits typically spring up at the early stage of a recovery led by strong gains in labor productivity that are not matched by raises in real wages. On the other hand, profits tend to decline in the later stages of an expansion as costs start rising faster than revenues, reducing profit margins. This is often accompanied or even caused by a tightening of labor market conditions that pushes up labor costs, cuts down profits, and as a consequence leads to a reduction of investment activity (again see Zarnowitz [1992] for a detailed analysis).

It is our belief that some of these facts can be accounted for by removing the Walrasian market-clearing mechanism from the labor market and by replacing it with an explicit model of labor relations. In this paper we begin to do so by assuming that contractual arrangements allocate labor resources in a manner that exploits the gains from trade that result from workers' difficulty in shedding cyclical income risk and entrepreneurs' (assumed) higher tolerance for such risk. The theoretical underpinnings of this approach go back to the seminal works of Baily (1974) and Azariadis (1975), which were based on the idea that labor markets embody an insurance aspect where labor's claims on output are partially fixed prior to the realization of output and entrepreneurs bear a disproportionate share of the output uncertainty.

In exchange for this provision of income insurance to workers, entrepreneurs gain a more flexible labor supply. As stated with great clarity in Rosen (1985), "Contractual income transfers smooth consumption, which interacts with labor utilization by eliminating income effects. The prominence of substitution effects promotes an elastic labor utilization response to socially diversifiable external shocks. *Contracts tend to increase the volatility of employment*" (p. 1145). Consequently, an interpretation of the present work that we wish to stress is that it allows for significant observable intertemporal substitution, consistent with the empirical evidence in Hall (1988), even when parameterization of workers' intratemporal labor supply elasticity (elasticity of substitution between consumption and leisure) is constrained by the available microeconomic evidence.

This approach is based on the joint hypotheses that employees are more risk averse than employers and that they cannot access financial markets to independently achieve intertemporal consumption smoothing to the extent that employers can. The first hypothesis is somewhat arbitrary, at least on strict empirical grounds. While there are well-known theoretical justifications for its adoption (from Knight [1921] to Kihlstrom and Laffont [1983]), we lack hard empirical evidence to be used either against or in favor. In our research we have chosen to fix the entrepreneurs' risk aversion and to treat the workers' risk aversion as a "free parameter." The validity of this method can be judged only by the power of its predictions and by the extent to which "unreasonable" differences in risk aversion are needed to deliver interesting results. The numerical simulations presented in Section III show that we need relatively small differences in risk aversion to account for most of the empirical regularities we claim to explain.

The second hypothesis seems easier to defend. An almost endless array of studies on the distribution of wealth show a strong concentration in the upper tail of the population (e.g., Smith 1980; Atkinson 1983; Cowell 1984; Champernowne and Cowell 1990).

This is particularly true for financial wealth and for the ownership of equities. If one excludes pension funds (which are seldom, if ever, used to achieve cyclical consumption smoothing), the percentage of individuals who own and actively trade financial instruments in organized security markets is remarkably small. Mankiw and Zeldes (1991), for example, report strong evidence that no more than 25 percent of households engage in these types of activities. More important for our concerns is the fact that similar figures emerge from the literature on consumption smoothing and market incompleteness. For example, using aggregate data, Campbell and Mankiw (1989) find that an approximate 50-50 split occurs between households that satisfy the permanent income hypothesis and households that are constrained in their cyclical borrowing-lending possibilities. Results on micro-level data are more conservative. The cumulation of evidence presented in Hall and Mishkin (1982), Hubbard and Judd (1986), Mariger (1986), and Jappelli (1990) suggests a consensus view that 20 percent of U.S. families are liquidity constrained and behave in a manner that is inconsistent with the pure life cycle model.

Furthermore, daily observations suggest that a large portion of actual investment decisions is concentrated in the hands of a small fraction of agents. While this may be the outcome of some complicated arrangement solving an economywide principal-agent problem, we seriously doubt the realism of such an interpretation. It seems simpler and more realistic to assume that the few agents taking responsibility for investment decisions are providing insurance services to the remaining portion of the households, not by trading assets that households effectively own, but through the employment relation.

In the model below, two types of individuals meet in each period: workers (proletarians) and entrepreneurs (capitalists). Before uncertainty is realized, the entrepreneurs offer to the workers a contract specifying the hours of work and the total payment they will receive in each possible future state of the world. Once the contract is mutually agreed on, both agents will stick to it, thereby assuming away the ex post recontracting and enforceability issues arising in the optimal contract literature (see Hart and Holmström [1987] for a recent survey and discussion).

The workers consume in each period all their wage payments, whereas the entrepreneur (who also supplies a portion of the total work effort) acts like the usual infinitely lived intertemporal maximizing representative agent. Capital accumulation decisions, in particular, are still modeled along the lines of Brock and Mirman (1972) as implemented in the RBC tradition of Kydland and Prescott (1982) and Long and Plosser (1983).

A typical cycle in our model consists of the following stages. Begin near the end of a recession period, when the economy has been hit by a sequence of negative shocks. Before the positive shock is realized, workers' expected utility from selling their time on tomorrow's spot market is low. This induces a low reservation utility and, consequently, a contract specifying a wage-labor combination that fixes the wage in future good states well below the marginal productivity of labor. When a positive shock is realized, entrepreneurs reap most of the benefits from the higher labor productivity. The contract also specifies a relatively high supply of labor in good states, and these two things jointly boost profits and therefore investments. As labor productivity increases, so does workers' reservation utility, thereby affording them a stronger bargaining position. This generates contracts more favorable to workers that progressively erode profit margins, increase their own consumption, and, as the recovery progresses, also reduce the incentive to invest in physical capital. At the end of the boom, contracts reflect the tight labor market conditions and, when a negative shock arrives, will magnify its impact on the firms' profitability. In turn, this induces a sharp decline in profits and investments near the peak of the cycle when the contraction occurs.

It is important to stress that the introduction of a labor contract does not alter only the cyclical pattern of wages and hours but has an impact also on the way in which investments, profits, and the labor share respond to the exogenous shocks. Basically the employees "lend" to the employers in good periods and "borrow" from them in bad ones. This increases the oscillations of profits, which now bear a much larger portion of the shock in productivity. It also increases their correlation with output, and it should tend to create a negative correlation between labor share and output. Furthermore, profits are now the crucial source of funds for the new capital; hence one expects the volatility of investments to increase as well, which it does.

There have recently been other attempts to employ risk-sharing arguments in models seeking to explain macroeconomic fluctuations, most noticeably Danthine and Donaldson (1992) and Gomme and Greenwood (in press). A comparison between our methodology and those adopted by these authors is therefore appropriate.

The Danthine and Donaldson model is quite different from the one we use. Leisure does not enter utility functions, and workers are divided into two groups (young and old), with only the second being covered by a contract. The contract guarantees full employment to the old people; the young enter and exit the employment relation according to Walrasian demand but have their income protected through a minimum wage and unemployment compensation financed by a tax on profits. It is therefore unclear what is the role played by the labor contract in generating the model's high volatility of labor as it comes all from the young portion of the population. Also it is unclear whether workers' reservation utility varies along the cycle or is instead specified once and for all at the beginning of time. Danthine and Donaldson are successful in mimicking observed volatility in hours. On the other hand, they do not report wages, profits, and factor shares, so one cannot evaluate their model's performance along those dimensions.

The model studied by Gomme and Greenwood is closer to ours. The descriptions of the economy and of its technology and population are quite similar. In contrast to us, they specify preferences with an endogenously time-varying and agent-specific discount factor, whose impact on the equilibrium dynamics is hard to disentangle from that of the risk-sharing arrangement. A second, more relevant, difference is their treatment of the labor contract. Workers and entrepreneurs are both allowed to smooth consumption by holding financial securities in a complete market environment. The amount of borrowing-lending that employees carry out through securities is then included in the wage bill together with the usual marginal productivity payment. Consequently, the optimal contract is not studied directly, and there is no endogenous determination of the two parties' bargaining strength. More to the central point, following along the ideas of Wright (1988), Gomme and Greenwood's methodology assumes that the introduction of labor contracts will change only observed factor payments but will have no impact on the real allocations. The present paper is based on the opposite assumption, that is, that the non-Walrasian features of labor markets affect not only the denomination of factors' payments but also the intertemporal behavior of most aggregate variables.

The paper is articulated in three other sections. Section II describes the theoretical model and briefly examines the qualitative intuitions underlying our approach. Here we spend some time discussing possible alternative formulations of the contractual environment that give rise to different levels of bargaining power and relatively different allocations of cyclical risk. Section III specifies the adopted functional forms, derives the equilibrium relations, and illustrates the outcomes of our simulations. In each case sample statistics are reported and compared to the relevant ones for the U.S. data during the postwar period. Section IV concludes the paper and discusses some of the issues that are still left open.

II. The Theoretical Framework

We study the following environment. There are two kinds of infinitely lived agents: those that own some stock of capital and those that do not. For each type, a continuum of identical individuals is present. We assume that there are $m \geq 1$ proletarians for each capitalist. Individuals of type 1 are born without any stock of capital and are more risk averse than their type 2 capitalist counterparts. People that are not shareholders are prevented from accessing capital markets to borrow/lend out of their labor income. This constrains their consumption and wage payments to coincide in each period.

Capitalists instead can borrow and lend at will in a perfectly competitive capital market. In each period, after observing a realization of the technology shock S_t, they organize the production process, pay the workers, and retain the residual output to be either consumed or invested in future capital stock.

There also exists a competitive market for θ-period-ahead labor contracts ($\theta \geq 1$, with θ an integer), where, at the end of each period, shareholders hire a fraction $1/\theta$ of next period's employees by offering them a menu $\{W(S), L(S)\}_{s\in\mathcal{S}}$ of possible salaries (or wage bills) and hours of work. A different pair $(W(S), L(S))$ is associated with each possible realization $S \in \mathcal{S}$ of technology shock. These contracts are assumed to be perfectly enforceable at no observable cost to either party.

The production function is written as

$$Y_t = S_t F(K_t, N_t, L_t),$$

where L_t is the labor supply of proletarians and N_t is the labor supply of the stockholders. The function F is standard: homogeneous of degree one, concave, monotone increasing, and smooth as needed. The technology shock S_t follows a stationary Markov process summarized by the transition function $P(S, S')$ with compact state space $\mathcal{S}$. Denote with $\mathcal{K}$ the real interval of feasible values of the capital stock.

Utility functions are denoted by $v(\tilde{c}, T - L)$ for agent 1 and $u(c, T$

$-N)$ for agent 2. We want to assume that agent 1 is more averse to consumption risk than agent 2, which means

$$\frac{-v_{11}(\tilde{c}, T-L)\tilde{c}}{v_1(\tilde{c}, T-L)} > \frac{-u_{11}(c, T-N)c}{u_1(c, T-N)}$$

for $\tilde{c} = c$ and $N = L$. The common intertemporal discount factor is denoted by $\delta \in (0, 1)$.

A. *Equilibrium without Contracts*

To compute the proletarians' reservation utility when bargaining over the labor contract, we need to look first at the competitive equilibrium when the two parties can only trade spot. In this case, after the shock S_t has been observed, agent 1 sells labor on the spot market, and agent 2 buys it.

To avoid confusing individual choices with equilibrium outcomes, we shall use lowercase letters to denote the former (i.e., l for agent 1; n, k, and c for agent 2) and capital letters to denote the latter (L, N, K, and C).

For an agent of type 1, labor supply is the solution to the simple problem

$$\max v(\tilde{c}_t, T - l_t)$$
$$\text{subject to } \tilde{c}_t \leq W_t = w_t \cdot l_t.$$

The first-order condition characterizing this choice reduces to

$$\frac{v_2(w \cdot l, T-l)}{v_1(w \cdot l, T-l)} = w, \tag{1}$$

which under the usual nondegeneracy conditions gives a labor supply function $l_t = l^s(w_t)$.

The stockholder solves a more complicated problem. Given a pair of initial conditions (S_0, k_0) and a stochastic sequence of wage rates $\{w_t\}_{t=0}^{\infty}$, he has to choose his own labor supply n_t, the amount of labor l_t^d he demands from each of the m agents of type 1, his consumption level c_t, and his investment level $i_t = k_{t+1} - (1-\mu)k_t$ for all periods $t = 0, 1, \ldots$. His stochastic optimal control problem and associated value function can then be written as

$$W(S_0, k_0) = \max\left\{\sum_{t=0}^{\infty}\delta^t \int_{\mathcal{S}} u(c_t, T - n_t) P_t(S_t, dS_{t+1})\right\}$$
$$\text{subject to } c_t + k_{t+1} = S_t F(k_t, n_t, ml_t) + (1-\mu)k_t - w_t \cdot ml_t. \tag{2}$$

Transversality condition aside, this yields the following array of necessary and sufficient first-order conditions, where λ_t denotes the Lagrange multiplier associated with the resource constraint:

$$u_1(c_t, T - n_t) = \lambda_t, \tag{3a}$$

$$u_2(c_t, T - n_t) = \lambda_t S_t F_2(k_t, n_t, ml_t), \tag{3b}$$

$$S_t F_3(k_t, n_t, ml_t) = w_t, \tag{3c}$$

$$\delta^{-1}\lambda_t = \int_{\mathcal{S}} \lambda_{t+1}\,[S_{t+1}F_1(k_{t+1}, n_{t+1}, ml_{t+1}) + (1 - \mu)]P(S_t, dS_{t+1}). \tag{3d}$$

A spot equilibrium is then obtained in two steps: first substitute the labor supply function $l^s(w_t)$ in place of l_t in (3) and in the resource constraint underlying (2) and impose market clearing in the consumption and capital good markets. Then solve the system of equations (3) to yield a set of functions $\{w(\cdot), L(\cdot), N(\cdot), C(\cdot), \tau(\cdot)\}$ depending on the state variables $Z_t = (K_t, S_t)$ and such that (*a*) $ml^s(w(Z_t)) = L(Z_t)$ solves (1) for all $t = 0, 1, \ldots$ and (*b*) $c_t = C(Z_t)$, $n_t = N(Z_t)$, $ml_t = L(Z_t)$, and $K_{t+1} = \tau(Z_t)$ solve the programming problem (2) given $w_t = w(Z_t)$.

B. *Equilibrium with Contracts*

Begin by defining agent 1's reservation utility at time t. This is the minimum total utility over the lifetime of the contract he will accept at time t when signing a contract for the θ periods $t + 1, \ldots, t + \theta$. It will be denoted as $\bar{v}_t$. It depends on the state of the economy at the end of period t and on the expectations it induces about future states. We can formally write it as

$$\begin{aligned}\bar{v}_t &= E_t\left\{\sum_{i=1}^{\theta} v(\tilde{c}_{t+i}, T - l_{t+i})\delta^i \,\middle|\, Z_t\right\} \\ &= \sum_{i=1}^{\theta} \delta^i \int_{\mathcal{Z}} v(\tilde{w}(Z_{t+i}) \cdot l^s(\tilde{w}(Z_{t+i})), T - l^s(\tilde{w}(Z_{t+i})))Q_i(Z_{t+i-1}, dZ_{t+i}),\end{aligned} \tag{4}$$

where $\mathcal{Z} = \mathcal{S} \times \mathcal{K}$ denotes the set of feasible pairs (K_t, S_t) and $Q(Z, dZ')$ is the equilibrium transition function (see Stokey, Lucas, and Prescott [1989] for details). Furthermore, in (4) the notation $\tilde{w}(\cdot)$ indicates the equilibrium wage as a function of the state Z when all workers but one have entered a contractual arrangement. This is the spot market salary that an individual worker should expect if he does not accept the employer's offer but all the other m/θ workers do. It will correspond to the marginal productivity of the input L evaluated at the level of $L(Z_{t+i})$, which is prescribed by the contract and will be determined below. The function $l^s(\cdot)$ is instead the individual labor supply function derived in (1).

When offering a contract, the stockholder must take into account the expected utility constraint induced by the workers' option of switching to the spot market and therefore obtaining at least $\bar{v}_t$. How much utility the nonstockholder should expect from the contract depends on relative bargaining powers. In this paper we take as a benchmark the case in which the proletarians have no bargaining power and all the gains from trade are collected by the capitalists. Obviously this is not completely realistic, but we believe that allowing more bargaining power to the workers would not substantially change the relative variability of wages and hours. We suspect, though, that it might have nonnegligible effects on the cyclical behaviors of capital and labor shares.

The stockholder decision problem can be described along the following lines. Given the state of the system at the end of period t, $Z_t = (K_t, S_t)$, and conditional on his choice of future capital stocks k_{t+i}, he needs to offer a contract $\{W(Z_{t+i}), L(Z_{t+i})\}_{i=1}^{\theta}$ to his prospective workers and simultaneously make contingent plans as to what kind of consumption levels $c(Z_{t+i})$, labor efforts $n(Z_{t+i})$, and investment $i(Z_{t+i})$ he will carry out. While the overall equilibrium values have to be determined at once, here we can examine the two problems separately. Let us begin with the contract design problem.

The implicit contracts literature (see Rosen [1985] for a survey) teaches that the crucial properties of the optimal arrangement depend on the assumptions one is willing to make on the different degrees of risk aversion of firms and workers, on the nature of the available information (public vs. private), and, in certain circumstances, on the income elasticity of leisure for the nonshareholder. This extreme sensitivity of the optimal contract generates a large number of outcomes that serve no purpose in the present investigation and would be very hard to follow in any case.

From our viewpoint, the salient feature of a contract is that it provides workers with an insurance mechanism during bad periods and entrepreneurs with a source of funds during good periods. This property is shared by both public and private information contracts. The latter is especially relevant only in the study of over- and underemployment of workers in (respectively) good or bad periods, a topic that does not concern us here (see Chari 1983; Green and Kahn 1983). Given that the computational complexity implied by the asymmetric information model is orders of magnitude higher than the one implied by the public information setup, we have restricted our present analysis to the latter. To maintain the analytical treatment within reasonable bounds, we also concentrate on the special case of one-period-ahead contracts (i.e., $\theta = 1$) and leave the exploration of the impact of staggered multiperiod contracts for future work (see Horvath 1994*b*).

When the realization of the shock is public information, wages and employment can be made conditional just on S. A contract is then a pair of functions $\{W(S), L(S) = m \cdot l(S)\}$ maximizing the capitalist's expected utility subject to the constraint that each type I agent has an expected utility no less than his reservation utility $\bar{v}_t$ as defined in (4). For the time being, let the equilibrium values of $C_{t+1}, N_{t+1}, K_{t+1}$, and K_{t+2} be taken parametrically by the capitalist. The optimal contract solves

$$\max_{w(\cdot),L(\cdot)} \int_{\mathcal{S}} u(c_{t+1}, T - N_{t+1}) P(S_t, dS_{t+1})$$

$$\text{subject to} \int_{\mathcal{S}} v(W(S_{t+1}), T - l(S_{t+1})) P(S_t, dS_{t+1}) \geq \bar{v}_t, \quad (5)$$

$$0 \leq c_{t+1} \leq S_{t+1} F(K_{t+1}, N_{t+1}, L(S_{t+1})) + (1 - \mu) K_{t+1} - K_{t+2} - m \cdot W(S_{t+1}).$$

It is well known (see, e.g., Hart and Holmström 1987) that the unique optimal contract is fully characterized by the following three conditions:

$$m \cdot u_1(C_{t+1}, T - N_{t+1}) S_{t+1} F_3(K_{t+1}, N_{t+1}, L_{t+1}) = \eta_{t+1} v_2(W_{t+1}, T - l_{t+1}), \quad (6a)$$

$$m \cdot u_1(C_{t+1}, T - N_{t+1}) = \eta_{t+1} v_1(W_{t+1}, T - l_{t+1}), \quad (6b)$$

$$\int_{\mathcal{S}} v(W_{t+1}, T - l_{t+1}) P(S_t, dS_{t+1}) \geq \bar{v}_t, \quad (6c)$$

where η_{t+1} is the Lagrange multiplier on the expected utility constraint, and the dependence of W and l on S_{t+1} has been omitted to economize on space.

The properties of the contract are straightforward and will not be repeated here. For our purposes it will suffice to stress that the risk-sharing condition (6b) is generally not satisfied by the spot equilibrium allocation. The contract in fact allows the entrepreneur one extra degree of freedom: the ratio between his marginal utility of consumption and the worker's marginal utility of consumption will now be equal to the constant η_{t+1} in all states, whereas in the spot economy that same ratio satisfies only

$$\frac{u_1(C_{t+1}, T - N_{t+1})}{v_1(W(S_{t+1}), T - l(S_{t+1}))} = \frac{u_2(C_{t+1}, T - N_{t+1})}{v_2(W(S_{t+1}), T - l(S_{t+1}))} \times \frac{F_3(K_{t+1}, N_{t+1}, ml(S_{t+1}))}{F_2(K_{t+1}, N_{t+1}, ml(S_{t+1}))},$$

which need not be constant with respect to $S_{t+1} \in \mathcal{S}$.

A second implication of (6) has to do with the sensitivity of $W(\cdot)$ with respect to S_t for any given K_t. As noted in Rosen (1985) for the case in which u is linear, only when workers' preferences are completely separable in consumption and leisure does the optimal contract predict that workers' and entrepreneurs' consumptions should be perfectly correlated across states of the world, whereas a nonseparable $v(\cdot, \cdot)$ links consumption behavior and the employment level of workers. In our own application the utility function is not linear, and we have not observed any relevant difference in this regard between the behavior of the separable model described below and that of a nonseparable version we have also simulated.

Denote by $W^*(\cdot)$, $L^*(\cdot)$ the equilibrium solution to (6) as a function of the state and of the other equilibrium variables. Under the assumption that all entrepreneurs are the same, competition in the market for contracts guarantees that in equilibrium the contracts will be identical across firms. The envelope theorem justifies our use of equilibrium notation when studying the dynamic programming problem of the representative capitalist:

$$U(S_t, k_t; W^*(\cdot), L^*(\cdot)) = \max_{n_t, c_t, k_{t+1}} \{u(c_t, T - n_t) + \delta \int_{\mathcal{S}} U(S_{t+1}, k_{t+1}; W^*(\cdot), L^*(\cdot)) P(S_t, dS_{t+1})\} \quad (7)$$

subject to $c_t + k_{t+1} \leq S_t F(k_t, n_t, L^*(\cdot)) + (1 - \mu)k_t - k_{t+1} - mW^*(\cdot)$.

Under standard restrictions (see, e.g., Stokey et al. 1989, chap. 9), (7) is known to possess a unique solution, summarized by the policy function $k_{t+1} = \tau(k_t; S_t, K_t)$. This function is continuous in k_t and K_t for any given S_t. A characterization of the (interior) optimal choices of the entrepreneur can be obtained by looking at the transversality condition and the first-order conditions

$$u_1(c_t, T - n_t) = \lambda_t, \quad (8a)$$

$$u_2(c_t, T - n_t) = \lambda_t S_t F_2(k_t, n_t, L^*), \quad (8b)$$

$$\delta^{-1}\lambda_t = \int_{\mathcal{S}} \lambda_{t+1}[S_{t+1}F_1(k_{t+1}, n_{t+1}, L^*) + (1 - \mu)] P(S_t, dS_{t+1}), \quad (8c)$$

where λ_t denotes once again the Lagrange multiplier associated with the technological constraint in (7).

A competitive equilibrium for the contract economy is then routinely defined by the existence of a set of functions $W^*(\cdot)$, $L^*(\cdot)$, $C(\cdot)$, $N(\cdot)$, and $\tau(\cdot)$ depending on the state vector $\mathbf{Z}_t = (S_t, K_t)$ and such that (*a*) $W^*(\cdot)$ and $L^*(\cdot)$ solve (5) for all $\mathbf{Z}_t$ given $C(\cdot)$, $N(\cdot)$, and $\tau(\cdot)$ and (*b*) $C(\cdot)$, $N(\cdot)$, and $\tau(\cdot)$ solve (7) for all $\mathbf{Z}_t$ given $W^*(\cdot)$, $L^*(\cdot)$.

C. *Bargaining Power*

The formulation given in (5) of the way in which the contractual agreements are reached implicitly assumes that all the bargaining power rests with the capitalists and that the proletarians walk away from the labor contract room with the same expected utility they carried when they walked in. One may indeed think of situations in which type 1 agents have some market power and are therefore able to obtain more than their reservation utility.

This need not destroy the efficiency properties of the optimal contract, which can be readily interpreted as the outcome of a Pareto-efficient allocation in which the two parties are given weights different from those implicit in (5). A simple way of formalizing this approach is to replace (5) with the following problem. Given the state vector $\mathbf{Z}_t = (S_t, K_t)$ and the equilibrium values of N_t and K_{t+1},

$$\max_{W,L} \int_{\mathcal{S}} \left[\nu_t u(C_t, T - N_t) + (1 - \nu_t) v\left(W, \frac{T - L}{m} \right) \right] P(S, dS')$$
$$\text{subject to } 0 \leq C_t \leq S' F(K_t, N_t, L) + (1 - \mu) K_t - K_{t+1} - mW. \quad (9)$$

The parameter $\nu_t \in [0, 1]$ is chosen arbitrarily, and it is a measure of the degree of market power of the entrepreneur. By varying ν_t between zero and one, we can trace out the whole expected utility possibility frontier. It is readily seen that when η_t in (5) is set equal to $(1 - \nu_t)/\nu_t$ in (9), the two problems become identical.

It is tempting to ask whether different choices of ν_t might have quantitatively relevant implications for the equilibrium behavior of the labor market variables. Taking our framework seriously yields an upper ($\bar{\nu}$) and a lower ($\underline{\nu}$) bound. The former is associated with guaranteeing that the solution to (9) provides the workers with the same level of expected utility they receive under the spot equilibrium, and the latter guarantees to the entrepreneurs their expected utility under the spot arrangements. An analysis along this line is not performed here. Unreported simulations suggest that, for reasonable values of ν, the results would be insignificantly different from those reported later in Section III.

We have also studied the behavior of our economy in the presence of a contractual arrangement under which the proletarians are guaranteed a *constant* level of utility in each future state of the world. This constant utility level has been chosen to be equal to their expected utility in the spot equilibrium. It is rather obvious that this contract is not optimal in the Pareto sense: both parties could be made better off by trading some uncertainty.

Let $\bar{v}_t$ be defined as in (4) above. Let $g(\bar{v}_t, l(S_t))$ solve

$$v(g(\bar{v}_t, l(S_t)), T - l(S_t)) - \bar{v}_t = 0. \tag{10}$$

The function $g(\cdot)$ always exists and is well defined under standard restrictions. The contractual problem replacing (5) can then be written as

$$\begin{aligned} &\max_{l(S_t)} \int_{\mathcal{S}} u(c_t, T - n_t) P(S_{t-1}, dS_t) \\ &\text{subject to } 0 \leq c_t \leq S_t F(k_t, n_t, ml(S_t)) \\ &+ (1 - \mu)k_t - k_{t+1} - mg(\bar{v}_t, l(S_t)). \end{aligned} \tag{11}$$

The optimal contract is fully characterized by the first-order condition

$$S_t F_3(k_t, n_t, ml(S_t)) = g_2(\bar{v}_t, l(S_t)) \quad \forall\, S_t \in \mathcal{S}. \tag{12}$$

With the obvious substitutions, the remaining choice variables of the entrepreneur and the equilibrium functions can then be determined as in subsection *B*.

Economic intuition and the formal results reported in Green and Kahn (1983) suggest that one should observe smaller fluctuations in L_t and larger fluctuations in W_t under the contract specified in (11) than under the optimal contract (5). As our simulations reveal, this is also the case in the fully parameterized model. Given that this is, on the other hand, the only way in which the introduction of the suboptimal contract affects the model economy, we do not report the results here.

III. The Parametric Models

In this section we introduce the specific functional forms utilized in the exercise and characterize the most intuitive properties of the equilibria.

The production function has been chosen to be Cobb-Douglas in capital (K) and total labor (E), which is a constant elasticity of substitution (CES) combination of proletarians' and capitalists' work efforts:

$$Y_t = S_t K_t^{\alpha} E_t^{1-\alpha},$$
$$E_t = [aN_t^{\rho} + (1-a)L_t^{\rho}]^{1/\rho}. \tag{13}$$

Here $L = ml$ is the total amount of proletarian labor employed. The parameters α and a are in the unit interval, and ρ is assumed negative to reflect the complementarity in production between the two types of labor.

The time-separable utility functions for both agents have been chosen from the CES class, under the restriction that the worker should be more risk averse than the entrepreneur. The entrepreneur has a utility function given by

$$u_t = \frac{1}{1-\psi} c_t^{1-\psi} + \frac{\gamma}{1-\psi}(T - n_t)^{1-\psi}. \tag{14}$$

As for the utility function of proletarians, we have experimented with both separable (in consumption and leisure) and nonseparable ones but observed very small and altogether insignificant differences for the behavior of the contract economy. We shall therefore report the result only for the separable version, which is

$$v_t = \frac{1}{1-\sigma} \bar{c}^{1-\sigma} + \frac{\theta}{1-\sigma}(T - l_t)^{1-\sigma}. \tag{15}$$

Obviously $\sigma > \psi$ is to be assumed throughout the rest of the paper. The technological shock S_t follows the stochastic process

$$S_{t+1} = S_t^{\rho_s} \exp(\zeta z_t), \quad z_t \sim N(0, 1), \tag{16}$$

with $\rho_s \in (0, 1)$ and $\zeta > 0$.

It should be noted that the utility functions specified are not consistent with balanced growth in output, consumption, and investment under exogenous productivity growth (see King, Plosser, and Rebelo 1988). We choose to abandon the class of functions nonseparable in consumption and leisure that are consistent with balanced growth because they yield an undesirable property: In the spot economy, worker labor hours are constant given that the worker consumes his income each period. Hours worked in the contract economy are little affected by the choice of functional class. However, relative to what obtains in the spot economy, the choice of nonseparable utility would make the contract economy seem too good for the wrong reason.

As in most RBC models, the functional forms assume that the intratemporal elasticity of substitution between consumption and leisure is equal to both the intertemporal elasticity of substitution between utility today and utility tomorrow and the elasticity of substitution across states of nature or one over the coefficient of relative risk aversion. It is apparent from microeconomic estimates of the relevant elasticity parameters (see Killingsworth 1983) that this may be an unrealistic assumption.

However, in light of our desire to compare the results of simulations from our model with those of previous RBC models, we proceed with the functional forms described above. The present model differs from the standard approach in that we allow the elasticities to vary across agent types, capitalist and proletarian, and we want to isolate the effect this has on the model's behavior. Adding additional degrees of freedom by enhancing the parameter space to allow for differences within agent types in the elasticities of substitution would be an interesting extension of the present analysis.

A. *Characterization of the Equilibrium*

The proletarians' labor supply under spot market conditions is

$$l^s(w) = \frac{T}{1 + \tilde{\theta} w^{1-(1/\sigma)}}, \tag{17}$$

where $\tilde{\theta} = \theta^{1/\sigma}$. Notice that $\sigma < 1$ is required to avoid a backward-bending labor supply function. Hence we shall always assume $0 < \psi < \sigma < 1$. The first-order conditions characterizing the solution to (7) are given by

$$C_t^{-\psi} = \lambda_t, \tag{18a}$$

$$\gamma(T - n_t)^{-\psi} = a(1-\alpha) S_t K_t^{\alpha} E_t^{1-\alpha-\rho} N_t^{\rho-1}, \tag{18b}$$

$$\delta^{-1}\lambda_t = \int_{\mathcal{S}} \lambda_{t+1}(\alpha S_{t+1} K_{t+1}^{\alpha-1} E_{t+1}^{1-\alpha} + 1 - \mu) P(S_t, dS_{t+1}). \tag{18c}$$

The optimal contract $\{W^*, L^*\}$ and the "bargaining power multiplier" η_t are computed by means of

$$\frac{m \cdot (1-a) MP_t \cdot L_t^{\rho-1}}{C_t^{\psi}} = \eta_t \theta (T - l_t)^{-\sigma}, \tag{19a}$$

$$\frac{m}{C_t^{\psi}} = \eta_t \cdot W_t^{-\sigma}, \tag{19b}$$

$$\begin{aligned} 0 = \int_{\mathcal{S}} [W_t^{1-\sigma} + \theta(T - l_t)^{1-\sigma} - W_{t,\text{spot}}^{1-\sigma} \\ - \theta(T - l_{t,\text{spot}})^{1-\sigma}] P(S_{t-1}, dS_t), \end{aligned} \tag{19c}$$

where the subscript "spot" indicates the equilibrium values associated with the labor supply function (17) and the notation MP_t stands for

$$MP_t = (1-\alpha) S_t K_t^{\alpha} E_t^{1-\alpha-\rho}.$$

Algebraic manipulation of the systems (18) and (19) yields useful insights into some basic properties of our dynamic contract economy. The total payments to an individual worker are

$$W^*(Z_t) = \left(\frac{MP_t \cdot L_t^{\rho-1}}{\theta}\right)^{1/\sigma} [T_1 - l^*(Z_t)]. \tag{20}$$

If we denote by w_{spot} the real wage of proletarians in the spot economy and by w the same real wage in the contract economy, it is easy to see that

$$\frac{w_{\text{spot}}}{w} = \frac{\theta l}{T - l}.$$

Hence during periods in which individual effort is higher than normal, the spot wage will tend to be above the contract wage, whereas the opposite occurs during periods in which l is below average. It is apparent from (19) that l is procyclical. A comparison of (19a) with the first-order condition determining the spot market labor supply function (17) shows that in the spot economy the level of employment reacts less to variations in its marginal productivity than in the contract economy because of the presence of a wealth effect that is altogether absent in (19a).

B. Parameterization

The system of equations we use to compute the dynamic equilibria of the model depends on a set of 13 parameters. Four pertain to the aggregate technology (α, ρ, a, μ), two are needed to specify the stochastic process for the technological shock (ρ_s, ζ), a group of five define the preferences of the agents (σ, θ, ψ, γ, δ), and the last two quantify the total time endowment and its distribution among capitalist and proletarians (T, m). Following along the methodology of Kydland and Prescott (1982), we shall now describe the numerical values we used and the empirical support for our choices.

For some of them the restrictions imposed by our model are indistinguishable from those imposed by the standard RBC models. Finding nothing objectionable in the standard calibration procedure, we have just adopted those same values. This choice sets $\delta = .993$, $\mu = .028$, and $T = 1{,}369$, which is the total number of nonsleeping hours per average person, per quarter.

The calibration of the remaining technology parameters is not a completely straightforward matter. The problem originates from our definition of the labor input E as a CES combination of the two types of time efforts, L and N. Unfortunately, we lack independent observations on these two variables. We considered for a moment the hypothesis of adopting the classification supervisory versus nonsupervisory work as a possible empirical proxy. Nevertheless, we chose not to consider this source of information on the ground that it provides a very bad and narrow representation of those aggregates to which our model refers. Gomme and Greenwood (in press) faced a similar problem, and we share their agnostic conclusions. The most reasonable option is therefore to treat total hours as a measure of E and proceed along.

With this caveat and the chosen values of δ and μ, one can proceed at estimating the technology parameter α independently from ρ and a. We have applied standard generalized method of moments (GMM) procedures to the orthogonality restriction induced by the Euler condition (18c), which uniquely depends on α (see App. A for data sources). Our point estimate $\alpha = .26$ differs substantially from the value of $\alpha = .36$ usually adopted in the RBC literature, but most of the difference seems attributable to our choice of the percentage change in the Standard & Poors 500 index as an instrument for the entrepreneurs' marginal rate of intertemporal substitution in consumption. As the appropriateness of this choice is predicated on the empirical relevance of the consumption-based capital asset pricing model and this model is at least debatable, we have also simulated our model with $\alpha = .36$, and the sample statistics turn out to change only slightly. To avoid giving the impression that our results depend on this particular estimate, we have used an average between the two values; that is, for the baseline model we have set $\alpha = .31$. To facilitate comparison, we have also chosen to report the outcomes of our simulations for both $\alpha = .26$ and $\alpha = .36$ in Appendix B.

As for the substitutability parameter ρ, lacking compelling empirical evidence on the matter, we have nevertheless found acceptable the idea that entrepreneurs and their employees are slightly complementary and not substitutable production factors, at least at the business cycle frequencies with which this study is concerned. Substitutability requires ρ to be negative but not too much so, and we have experimented with a few values in the interval $[-1.0, -.1]$ without noticing any relevant impact on the final outcomes. Very bizarre results obviously can be obtained at extreme values of ρ when the degree of complementarity between the two types of labor becomes exaggeratedly large.

Given that T has been set equal to 1,369, we next turn to the determination of how many proletarians are out there for each capitalist. The theoretical underpinnings of our framework together with the empirical evidence quoted in the Introduction suggest that somewhere between one-quarter and four-fifths of the population should be considered as composed of stockholders, implying that m lies in the interval [.25, 3]. However, we can restrict attention to a smaller set of plausible parameter values by contrasting the meaning of the parameter to our model with the intent of the empirical evidence. In our model, the number of workers relative to the total population is defined as the fraction of agents for whom consumption equals income. Compared with this definition, the definition of "liquidity constrained" used in the micro-level studies is not sufficiently inclusive since it counts as workers only those individuals who in the past have had credit denied to them. This understates the number of "workers" in the economy if there exist individuals who have not had consumption loans denied to them but nevertheless consume all their income each period. On the other hand, the evidence in Mankiw and Zeldes (1991) may be too inclusive with respect to what our model is trying to capture since they count as entrepreneurs only people who own stocks in publicly traded corporations. This overstates the number of "workers" if there exist people who do not own stocks in corporations but, nevertheless, own buffer stocks of capital in the form of houses, cars, privately held corporations, and so forth.

Campbell and Mankiw (1989) present empirical evidence that suggests that the fraction of workers whose consumption growth follows income growth is around one-half. In a stochastic environment we may not expect that workers (in the sense of the model's definition) are always able to consume their income every period. This motivates us to accept a definition of workers as individuals for whom consumption growth follows income growth at business cycle frequencies, and for the purpose of parameterizing the model, we focus on the macroeconomic evidence in Campbell and Mankiw (1989). To satisfy our critics, we also perform sensitivity checks to assure that any positive results are not achieved through critical parameter choices. While results seem to change little as $.5 \leq m \leq 2$, a number of sample statistics become very sensitive for values of $m > 2$ or $m < .5$. For this reason and also in order not to bias our calibration too heavily toward the hypothesis that a very large portion of the population is the worker type, we have chosen the value $m = 1.5$ for our baseline model.

Once a value of m is chosen, one can use income distribution data to fix the remaining technological parameter a. The idea is to choose a so that the steady-state portion of income going to the employees corresponds to the sample percentage of national income received by the bottom 60 percent of the population (the fraction 60 percent is implied by the choice of $m = 1.5$). Although the concentration of wealth evidenced in the data does not directly imply that it is only the poor who are credit constrained, the empirical evidence indicates a strong negative correlation between wealth and the presence of such constraints (see Attanasio 1994). Whether causal or not, this evidence has motivated us to specify a model in which the poorer group does not own capital stocks. Therefore, this is the manner in which we must interpret the evidence on the distribution of wealth, in the absence of any micro evidence on the distribution of wealth for the two types of individuals.

Depending on measurement techniques and various possible definitions of income, the values we have found in the literature for the percentage of income accruing to the bottom 60 percent of the population range between .30 and .36. As a point estimate we have chosen .33, which is the value reported for the United States in World Bank (1993, p. 297). In our model, though, the steady-state income distribution is also affected by the degrees of risk aversion of the two agents and by the intensity of their preferences for leisure. A reasonable choice of a must therefore be made jointly with that of the preference parameters, to which we move next.

Two of them (θ and γ) can be calibrated so that the model deterministic steady state satisfies some empirical restrictions on the typical fraction of total nonsleeping hours that individuals allocate to market activities. It is customary in the business cycle literature to use point estimates between .25 and .33 for this fraction, which in general require values between .9 and 1.3 for the model's parameters. As for σ and ψ, they are in some sense "free" in our model and are meant to capture the extent to which workers are more risk averse than entrepreneurs. After experimenting with a few nonextreme values, we have observed that relatively little variations occur for σ between .3 and .9 and ψ between .2 and .6. It should be noted that in our framework a value of one is in any case an upper bound for both degrees of risk aversion since larger values would imply a backward-bending spot labor supply function, hardly a realistic feature at the business cycle frequencies we are interested in studying.[2]

Still this leaves us with a large set of parameter values from which to make our choice. To restrict it further we have concentrated on two particularly important sample statistics: the correlations between wages and output and between consumption and output. The U.S. data reported in the Introduction suggest a low value for the former and a relatively high value for the latter. Sensitivity analysis shows that in our model their behavior depends in a nonlinear fashion on the

[2] Previous RBC models have typically specified a degree of risk aversion larger than unity, motivated largely by the inconsistency of lower degrees of risk aversion with empirically observed high levels of intertemporal substitution. For the reasons described above, the contract economy does not require excess risk aversion to match this feature of the data.

choice of a, σ, and ψ (varying θ and γ appropriately in order to match the sample statistics on the percentage of total hours spent at work).

In order to characterize such dependence, begin by considering figure 1, which reports the real wage standard deviation as a fraction of the output standard deviation for different values of a, σ, and $d = \sigma - \psi$. The height of the surfaces in the graphs, read off the vertical axis, corresponds to the ratio of σ_w to σ_y calculated by simulations of the model for varying values of the parameters σ, d, and a.

The ratio first decreases and then increases in d, with the location of the minimum points occurring at higher values of d as σ and a increase. The figure suggests that the smoothest wages occur not when the amount of insurance desired by the workers is extremely high but instead when it is moderately high. Furthermore, as the workers become more risk averse, the smoothest wages occur when their relative bargaining position worsens (higher d). Finally, as a increases, the volatility of wages relative to output becomes more sensitive to the value of σ and less sensitive to the value of d.

To explain the convex shape of the surfaces in figure 1, consider figure 2, which shows that the relative volatility of hours is nearly linearly increasing in d but linearly decreasing in σ. Recall that the contract tends to smooth out W_t, the total wage bill, and that the aggregate real wage is obtained by averaging W_t/L_t with the marginal productivity of the entrepreneurs' hours. As d increases, the volatility of L_t increases as it becomes more correlated with output. This tends to compensate for the correlation of W_t with output, thereby reducing the volatility of w_t with respect to that of output. But as d increases, the volatility of W_t also keeps increasing until it outdoes that of L_t, thereby pushing up the relative volatility of w_t again. This logic implies that at low and increasing values of d, the real wage should be more highly correlated with output than at very high values of d. This is confirmed in figure 3, where wages are less correlated with output as d increases, as σ decreases, and finally as a decreases.

In figures 3 and 4, we have reported the simulated correlations between w_t and y_t and between c_t and y_t as mesh surfaces above the σ, d plane. The outlined planar surfaces drawn in all the figures indicate the estimated values for the statistics from the U.S. data sample. In both sets of figures the mesh surfaces slice through the planes denoting the empirical point estimates for a range of values for σ and d. Note that for values of $a = .46$, one can get close to both planes for choices of $\sigma = .32$ or $.34$ and $d = .1$ or $.12$. Further simulations (not reported) show that this is the case for even lower values of a. These findings have led us to set our baseline parameter values equal to $a = .46$, $\sigma = .32$, and $\psi = .22$. As we mentioned before, without direct observations, the reasonableness of these

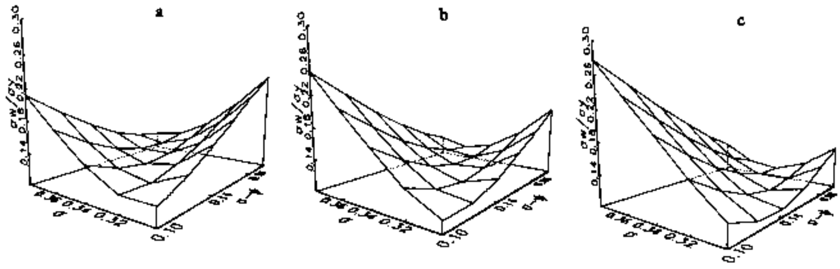

FIG. 1.—Sensitivity of σ_w/σ_y. *a*, a = .46; *b*, a = .50; *c*, a = .54

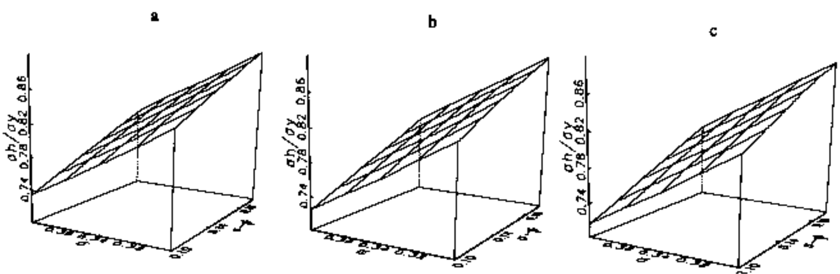

FIG. 2.—Sensitivity of σ_h/σ_y. *a*, a = .46; *b*, a = .50; *c*, a = .54

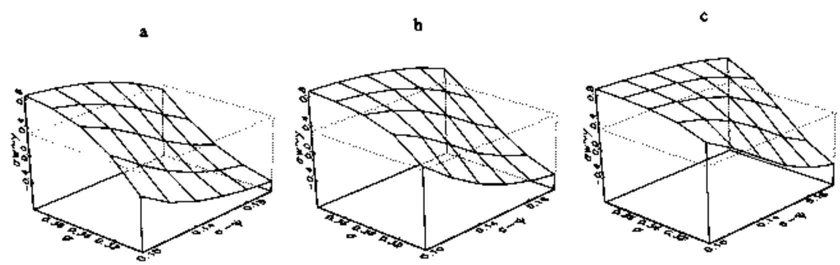

FIG. 3.—Sensitivity of $\sigma_w \sim y$. *a*, a = .46; *b*, a = .50; *c*, a = .54

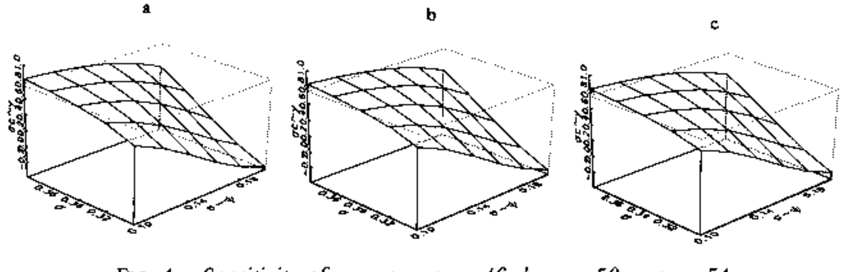

FIG. 4.—Sensitivity of $\sigma_c \sim y$. *a*, a = .46; *b*, a = .50; *c*, a = .54

choices can be judged only ex post by the quality of the overall model's performance. On a priori grounds, we find it perfectly acceptable.

Finally, the two parameters of the stochastic process S_t have been estimated by constructing a "Solow residual" series in the ordinary way. This series has been used to compute GMM estimators for the autocorrelation parameter ρ_s, and ζ has been obtained by applying GMM to the orthogonality restriction on the innovations of S_t. This procedure gives the two values $\rho_s = .968$ and $\zeta = .010$.

While these are typical of parameter values used in other RBC simulation exercises, it was noted above how the dynamic response of the contract model is heightened by the existence of the contract between workers and entrepreneurs. Therefore, the same size innovations to log productivity will have a larger instantaneous impact in the contract economy than in its spot counterpart. Alternatively, the contract model requires smaller shocks than the spot economy to match the volatility of output. The interpretation of a Solow residual innovation in the contract model is the same as in the standard RBC literature: an exogenous change in the productivity of all factors of production. The interpretation of the output innovation that follows the productivity innovation is different. The output innovation is composed of two parts: one associated with the change in productivity and the other associated with the contractual nature of worker-firm relationships.

C. *Simulation Results*

Using the set of parameter values listed in table 2, we have generated 100 samples of artificial economies with 180 observations each. The data so obtained were passed through the Hodrick-Prescott filter, and the results were averaged over the 100 samples.

Most of the results reported below appear quite robust to parametric variations and are very indicative of the ability of the model to capture some of the business cycle puzzles we discussed in the Introduction. In particular, three claims we have made seem to be consistent with the behavior of this artificial economy: (1) introducing a contract increases the volatility of hours and decreases that of real hourly wages; (2) the volatility of aggregate output is increased together with those of profits and the labor share (the last two also display the correct sign for output correlation); and (3) the correlation of wages and output can be reduced to almost zero (in fact at other acceptable parameter values it turns out to be slightly negative), whereas hours remain strongly correlated with output.

The performance of the model is also encouraging with respect to the first-order autocorrelation of the aggregate variables, but not

TABLE 2

BASELINE VALUES OF CALIBRATION PARAMETERS

$\delta = .993$	$\sigma = .32$	$\psi = .22$	$\gamma = 1.075$	$\theta = 1.195$	$\mu = .028$
$\alpha = .31$	$\rho = -.7$	$a = .46$	$\rho_s = .968$	$\zeta = .01$	$m = 1.5$

TABLE 3

BASELINE MODEL

Series	Standard Deviation	Correlation	Autocorrelation
Output	2.82	1.00	.725
Consumption	.94	.71	.607
Investment	8.71	.93	.728
Hours	2.28	.97	.739
Average labor product	.56	.94	.657
Real wage	.31	.41	−.084
Labor share	.50	−.88	.798
Profits	3.87	.96	.778

entirely so. As table 3 shows, the high persistence that characterizes the real wage in the post–Second World War data is not displayed by our model under Hodrick-Prescott filtering. If one takes into consideration the asymptotic standard errors of the sample estimate, the hypothesis that this autocorrelation is actually zero cannot be rejected at conventional significance levels. In any case, even a zero autocorrelation remains a far cry from the empirically observed values. Similar statistics are usually not provided for standard RBC models, but simulations we have run using a standard RBC model show that these features are common to both frameworks.

On the other hand, one should stress that the lack of persistence in real wages is relatively easy to eliminate in the contractual framework. It is induced by the fact that our contracts are only one period ahead and do not link workers and entrepreneurs for more than one quarter. This enables the two parties to quickly incorporate changes in aggregate productivity in the calculation of labor compensations.

Indeed, this is a very unrealistic feature of the model, which we have chosen to maintain here only because it greatly simplifies the numerical computations. Simulations of a simplified version of the contract model allowing for staggered multiperiod contracts lasting two to four quarters are presented in Horvath (1994*b*). They show that this modification loosens the short-run relation between changes in marginal productivity and real wages, resulting in a positive autocorrelation of measured real wages and, consequently, of the consumption series.

The model performs quite well in all the other dimensions, and when standard errors are taken into account, the empirical sample estimates (with the noted exception of the real wages autocorrelation coefficient) belong to the confidence intervals generated by the artificial economy and vice versa. Results are even stronger when the model and data are rendered stationary with a log-linear trend with a single breakpoint in 1973:1. Simulation results for log-linear detrending and first-differencing filters are available on request.

A quantitative feeling of the way in which the optimal contract affects the performance of the artificial economy can be gauged by comparing the sample statistics for the contractual model with those of the spot economy. This is done in table 4 for the standard deviation, output correlation, and unconditional first autocorrelation of the Hodrick-Prescott filtered data. All parameter values are the same as in table 3.

We finally compare the contract model and the Hansen (1985) "indivisible labor" model, which is correctly regarded as the paradigmatic RBC model of labor market behavior (see also Rogerson [1988] for the theoretical background). Hansen did not consider factor shares, nor autocorrelation coefficients. Table 5 is constructed accordingly. The parameter values for our model are those of tables 3 and 4.

It is fair to conclude that there is no visible dimension along which the contract model performs worse and some obvious and very important dimensions along which it performs better.

IV. Conclusions

We have shown that introducing simple forms of contractual labor relationships in a standard stochastic optimal growth model makes it display more realistic properties than those that obtain when the labor market is modeled in a purely Walrasian fashion. Wages and hours oscillate at the right magnitude and in the right direction without the need to introduce an unreasonably elastic labor supply function or "unobservable" institutional mechanisms. Factor share cyclical variability and correlation can be accounted for by the same contractual argument that also provides an explanation for the observed behavior of profits and investments at the peak and trough of the trade cycles.

Factor share oscillations, while going in the right direction, are still relatively small in our model. This is especially true for profits. This seems harder to capture: it may require moving away from a Cobb-Douglas specification for the aggregate technology as well as from the one-sector representation. Two-sector Cobb-Douglas models already allow for cyclical variations in factor shares; it remains to be seen

TABLE 4

CONTRACT VS. SPOT ECONOMY, HODRICK-PRESCOTT FILTER

	Contract Model			Spot Model		
Series	Standard Deviation	Correlation	Autocorrelation	Standard Deviation	Correlation	Autocorrelation
Output	2.82	1.00	.725	2.71	1.00	.701
Consumption	.94	.71	.607	1.23	.71	.870
Investment	8.71	.93	.728	11.57	.92	.675
Hours	2.28	.97	.739	2.11	.96	.690
Average labor product	.56	.94	.657	.63	.94	.752
Real wage	.31	.41	−.084	.68	.95	.740
Labor share	.50	−.88	.798	.00	.00	.000
Profits	3.87	.96	.778	2.71	.97	.701

TABLE 5

CONTRACT ECONOMY AND HANSEN (1985) ECONOMY

	Contract Model		Hansen (1985) Model	
Series	Standard Deviation	Correlation	Standard Deviation	Correlation
Output	2.82	1.00	1.76	1.00
Consumption	.94	.71	.51	.87
Investment	8.71	.93	5.71	.99
Hours	2.28	.97	1.35	.98
Average labor product	.56	.94	.50	.87
Real wage	.31	.41	. . .	. . .
Labor share	.50	−.88	. . .	. . .
Profits	3.87	.96	. . .	. . .

whether they are quantitatively relevant. Along these lines, one may also consider further departures from the purely competitive framework, such as the introduction of borrowing contractual arrangements between entrepreneurs and financial institutions.

It should be stressed that the borrowing constraints imposed on the workers in the model are extreme. However, the behavior of the model would be similar if workers were allowed to borrow on expectations of future income streams up to a limit, as long as the limit was a binding constraint in each period.

This line of research could ideally lead us to be able to dispense with the notion of large and frequent aggregate technological shocks. They are very vague and hardly measurable entities, which can be identified only after the fact by accepting uncritically a number of simplifications on the form of the production function and on the way in which inputs are rewarded. The theory of the business cycle that stems from dynamic general equilibrium models does not require aggregate shocks, neither from a logical nor from an empirical point of view as demonstrated in Horvath (1994*a*). Their current adoption seems to be motivated almost essentially by practical considerations: lacking endogenous sources of instability and built-in magnifiers, one has to resort to aggregate exogenous stimuli "to get things going." Further investigation in this area may well point to other endogenous sources of business fluctuations.

Another natural extension is to look at the asset pricing implications of the contractual approach. Results obtained with a model in which nonstockholders are the only suppliers of labor effort are quite promising. Intuitively this is due to a couple of factors. On one hand, as our model shows, profit earners now bear a much larger portion of the aggregate risk: return on equities is both much higher and more correlated with aggregate output. On the other hand, the equilibrium prices of assets are not evaluated by using aggregate consumption to compute the relevant rate of intertemporal substitution. Instead it is the consumption of stockholders alone that matters, and it need not be as stable and smooth as the economy's average consumption. In order to give operative content to this approach to asset pricing, one needs to be able to identify empirical measures of stockholders' consumption volatility.

Appendix A

Data Description

We have attempted to present statistics on and estimate parameters from data on private-sector, nonfarm, business production, and factor payments. To do so, we often begin with a broader category and subtract sectors that we do not wish to include (e.g., removing farm production from gross domestic product). In the list below, the series name is followed by the symbol that corresponds to the series in our model. A brief description of the data source is given with specific series abbreviations and, in some cases, additional notes.

Output: Y. Real gross national product less the production of farm, nonfarm housing, and government sectors, in 1982 dollars, reported quarterly in Citibase National Income and Product Accounts (NIPA): gnd82 − gpbf82 − gbuh82 − ggnp82. All data have been seasonally adjusted.

Consumption: $C + W$. Real total consumer expenditure on nondurables and services, in 1982 dollars, reported quarterly in Citibase NIPA: gcn82 + gcs82.

Investment: I. Real private-sector, fixed, nonresidential investment plus real expenditure on consumer durables, in 1982 dollars, reported quarterly in Citibase NIPA: gin82 + gcd82.

Capital stock: K. Stock of investment series constructed in the usual manner by comparing net and gross investment series. Source: Christiano and Eichenbaum (1990).

Total hours worked: $L + N$. Total hours worked in nonfarm, business sector (index = 100 in 1982), based on establishment surveys, reported quarterly by Bureau of Labor Statistics: LBMNU.

Real wages: $(W + MPN \times N)/(L + N)$. Hourly compensation of all nonfarm, business employees (index = 100 in 1982), based on establishment surveys, reported quarterly by Bureau of Labor Statistics: LBCPU, deflated by the variable defined below as price.

Profit: $Y - W - MPN \times N$. Nominal corporate profits before tax reported quarterly in Citibase NIPA: gpbt, deflated by the variable defined below as price.

Average labor productivity: $Y/(L + N)$.

Labor share: $(W + MPN \times N)/Y$.

Price: Implicit price deflator equal to nominal output series (NIPA: gnd −

gpbf − gbuh − ggnp) divided by the real output series described above under output.

Detrending methods: We induce stationarity by three alternate methods (only the results for Hodrick-Prescott filtering are reported in the text): a two-trend detrending procedure on the log levels of the data, the Hodrick-Prescott filter with λ, the cost of detrending in the filter's minimization function, set at 1,600, and log first-differencing. The last method is completely straightforward. The log-linear detrending allows for one trend in the log levels for 1947–72 and a potentially different trend for 1973–90. Naturally, the log-linear detrending removes less information than the Hodrick-Prescott filtering; however, questions remain whether the classical properties apply to the distribution of the log-linearly detrended series because they may still not be covariance stationary.

Appendix B

Alternative Parameter Values

To complete the description of the baseline model's performance we report next the sample statistics for the cases in which $\alpha = .26$ and $\alpha = .36$ (tables B1 and B2, respectively). All other parameter values are as in tables 3 and 4 with the following exceptions. In both cases, γ and θ have been adjusted to keep the appropriate ratio between working and nonsleeping hours. Also, in table B2 we have chosen slightly different values for a, σ, and ψ (holding $d = \sigma - \psi$ constant) in order to match the sample correlations between wages and output and between consumption and output. These new values are $a = .42$, $\sigma = .38$, and $\psi = .28$. No change of this kind was made for $\alpha = .26$ even if also in that case some very small variations of the same parameters would have allowed us to exactly match the named correlations.

TABLE B1

Contract Model ($\alpha = .26$, $\theta = 1.04$, $\gamma = .925$)

Series	Standard Deviation	Correlation	Autocorrelation
Output	2.92	1.00	.601
Consumption	1.33	.86	.552
Investment	10.22	.93	.587
Hours	2.26	.97	.612
Average labor product	.68	.95	.565
Real wage	.51	.88	.288
Labor share	.30	−.83	.655
Profits	3.68	.96	.661

TABLE B2

CONTRACT MODEL (α = .36, θ = .97, γ = .903)

Series	Standard Deviation	Correlation	Autocorrelation
Output	2.44	1.00	.720
Consumption	.71	.76	.489
Investment	6.03	.95	.727
Hours	1.91	.97	.741
Average labor product	.55	.95	.632
Real wage	.30	.40	−.128
Labor share	.50	−.89	.804
Profits	3.29	.96	.769

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